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Are large credit exposures a source of concentration risk?

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Abstract

Credit concentration risk may be the largest risk for a bank. The division into non-granular and granular portfolios based on portfolio size and individual credit sizes is essential for assessing the concentration risk of a portfolio. This paper identifies portfolio-specific large credits as a source of concentration and evaluates the effects of these to portfolios risks. Choices on risk measures and assumptions on portfolio structure are aligned with those on previous studies, which enables previous research to be complemented with results on large credits. The use of parameters and portfolio sizes from actual portfolios increases the applicability of the results. The results show that accumulating large equally--sized credits has both increasing and decreasing implications to concentration risk following initial portfolio structure. In the smallest portfolios the share of granularity exceeds 80% of credit risk. A novel risk-weight add-on formulation with a unified interpretation for banks using standardized methods or internal ratings-based method is also introduced.

Keywords: risk measurement, concentration risk, credit risk, Basel regulation

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1. Introduction

What are the effects of individual large credits to banks' credit risk? The reservation of capital to cover future losses from credit concentration risk is stated in banking regulations and potential losses from concentrations have a broad field of research. Definitions of the concentration risk generally separate risks related to credit size and sector. Within size-related risks, the division into non-granular and granular portfolios is essential for assessing whether the portfolio requires reserving capital from the perspective of a bank or a supervisor. Large credit exposures are a recognized dimension of the concentration risk with specific rules limiting the size of large credits for FI's. Large credits are also at the centre of definitions of risk by for example Skridulytė and Freitakas (2012). Yet, research on credit risk has a very limited contribution to the subject of large credits within portfolios of regulated institutions. For large credits there are no well-known results concerning the direction or magnitude of risk. This paper contributes to the literature on credit concentration risk by considering the contribution of large credits to credit risk. By choosing a methodology from the research of portfolio granularity and focusing on large credits, the research hypotheses on large credit are formulated.

Literature on credit concentration risk measurement covers simulation methods and analytical formulas to measure the concentration risk. Research in the area has also answered specific questions rising from the risk management and regulation perspective by aligning concentration risk measurement with relevant credit portfolio risk models. Credit concentration risk on large credits in the Basel regulations is implicitly part of regulation through setting large exposure rules and by requiring measurement on credit portfolio risks. Galaasen et al. (2020) approach the non-granularity of institution's portfolios in connection with large credits and find that large credits are an important part of actual portfolios even when the size of large credits is limited. While the methodology of concentration risk measurement would ideally be aligned with the Basel regime's institutional practices and the supervisor's requirements, there are still various approaches in this area. Instead of exact thresholds and unified practices, there are varying, and in some cases, undisclosed practices, as discussed by Grippa and Gornicka (2016). A somewhat comparable area of finance research where diversification is of interest is investment portfolios. In investment portfolio diversification the question of sufficient diversification in relation to the number of investments has an established mean-variance approach initially proposed by Markowitz (1952). For investment portfolios "rule of thumb" practices even exist with a decrease in ratio 1/n, as discussed by Brown (1976). Such rules are not presented for credit portfolios with heterogenous credit sizes and heterogenous risk parameters. For investment portfolio diversification there clearly are varying contributions too based on an individual investment's risks. For credit portfolios, the research on concentration has advanced to the use of measurement methodologies. The absence of general results on diversification size limits rises from the sensitivity of concentration risk to other credit portfolio risk measures. All generally used measurements on credit concentration risk decrease with the number of credits, but the rate of decrease and overall level of concentration risk are very sensitive to other risk features of the portfolio.

Banking regulation has rules both on concentration risk and on large exposures (BCBS 2014). The Basel regime's rules on large exposures set a limit on the size of credit exposure to a single counterparty of 25% of the FI's own funds. The indirect effect from own funds to credit portfolio has for example been simplified so that if the FI has own funds of 8% of its exposure, the largest credit could be 2% of all credit portfolio exposures (Hibbeln 2010). A case where a large exposure would exceed the regulation limit but would decrease the portfolio concentration risk may also be regarded as an unintended effect of regulation. The methodology for approaching large exposures as a portfolio risk ideally captures this potential unintended consequence and enables quantification of the effect. Having a large credit or large credits as an additional portfolio feature in comparison to a base case with variable portfolio size allows the treatment of large credits as an additional feature of the portfolio. The scope of this paper excludes very large portfolios. The selection of data restricts such portfolio sizes, that based on previous research for example by Hibbeln (2010) and Gordy and Lutkebohmert (2007), are in smaller size classes where there is an expectation of granularity risk remaining.

Several dimensions of credit portfolio regulation are considered. The capital requirement calculation in the Basel regime is aimed to be risk sensitive and it is divided into two pillars, identified as Pillar 1 and Pillar 2. Pillar 1 of the regulation gives exact formulas for the calculation of the capital requirement for certain risks and Pillar 2 requires analysis of certain remaining risks that were not calculated in Pillar 1. Furthermore, a third pillar in the regulation sets the transparency of risks through publications of risk levels for each FI. The methodology and data cover the internal model regime's Internal Ratings Based (IRB) approach in Pillar 1, research on the measurement methodology from Pillar 2 and grade level data published under Pillar 3. The use of established measurement methods and commonly used assumptions on portfolio structure allows comparison with previous research and enables previous research to be complemented with the results on large credits. Both analytical approximations and the Monte Carlo simulation are used in the measurement.

The paper is organized as follows. Chapter 2 gives a literature review for the risk of interest: granularity concentration in the context of the Basel regulations and chapter 3 derives the measurement methodology for the risk. Hypotheses are formulated at the end of chapter 3. Chapter 4 presents grade level data complemented with credit size assumptions. Chapter 5 presents the results of granularity adjustment as additional risk incorporated to an asymptotic single risk factor approach. Discussions and implications for future research are discussed in chapter 6.

2. Review on literature of credit portfolio granularity

The measurement of granularity in the credit portfolio may be divided into model-free and model-based methods. Model-free concentration risk measurement methodology has an established background in indicators which describe size-related features of a phenomenon. The Gini coefficient, introduced by Gini (1921), is a well-known measure of income inequality and applies to credit portfolio concentration risk as well. Several distance measures from base distribution, which in the context of credit portfolio is a size--homogenous credit portfolio, are introduced by Skridulytė and Freitakas (2012). A common feature for distance measures and the Gini-coefficient is that deviation from the size-homogenous portfolio does not reflect the overall portfolio size changes. For a given portfolio size n, the variability from different credit sizes is measured, but the overall change of n is not. A Herfindahl-Hirschman index (HHI), which was applied early, for example by Graham (1965), for measuring industry concentrations based on a firm's market shares, has a property of reflecting the total n. Like Gini, distance measures and HHI vary around values 0 to 1 and for all these a value of 0 describes the most diversified distribution and a value of 1 the most concentrated distribution. In the case of HHI and industry distribution, an example of the most concentrated industry is a single institution monopoly, and it receives a value of 1.

Model-based measurement of granularity links granularity to risk parameters and models. This area has advanced in research through the introduction of Pillar 2 concentration risk in the Basel regulations. The definition of risk has also evolved to imply regulatory features. Hibbeln (2010) classifies concentrations into name concentration, sector concentration and credit contagion. Skridulytė and Freitakas (2012) describe a total of 11 different types of credit concentration risks, including “large credits for single borrowers”. All 11 risks fall mainly into three categories in the classification of Hibbeln and following the Basel Committee’s early work on concentration risk (BCBS 2005a), Hibbeln also separates two types of name concentration risks: “single name concentration”, where a single counterparty is very large compared to other counterparties and “portfolio name concentration”, where a number of counterparties are small in relation to the size of the portfolio. Duellmann and Masschlein (2006) define concentration risk as a positive or negative deviation from a framework that doesn’t explicitly cover concentration risk. Although this definition through deviation is less specific in naming the risk, the definition has a key linkage to the banking regulation internal model approach and concentration risk measurement methodology has been extended in research specifically from this viewpoint. The regulations’ internal model regime on credit risk is based on the asymptotic single risk factor (ASRF) framework’s IRB formula. ASRF has two key assumptions: a single risk factor describes default correlations, and the second is that the portfolio is infinitely granular. However, calibration of the model in the regulation refers to well-diversified portfolios as different from completely diversified portfolios. Initially, the published granularity adjustment was suggested in combination with VaR 99.5% portfolio risk level while the actual Basel 2 regulation was implemented without granularity adjustment and increased VaR to 99.9%. In a specific line of research, the difference between Var 99.9% and Var 99.5% serves as an assumption of the concentration risk level covered by calibration. Gürtler, Heithecker and Hibbeln (2006) and Hibbeln (2010) present credit sizes which are sufficient to close this difference to an extensive combination of portfolio sizes, PD levels and asset correlations. Gordy and Lutkebohmert (2013) report results on granularity for practically observed portfolios. The portfolios have variable sizes and variable risk levels. According to the results, the GA variability is high in size class 1000–3999 credits and in smaller size classes, while for size class 4000–8999 GA already remains very low and stable. Hibbeln (2010) also tests the expected shortfall measures for corresponding risk levels and the parameter combination due to the limited coherence of the VaR measure. In several combinations of portfolio PD and asset correlations, a lower limit of 4000 credits is sufficient to state that the portfolio is diversified in 95% of combinations.

The development of granularity adjustment methodology was essentially initiated by Martin and Wilde (2002) and Gordy (2003), who developed analytical approximation for granularity as options for burdensome Monte Carlo simulations. This approach was essentially advanced by Gordy and Lutkebohmert (2013), who presented an improvement to granularity adjustment. The application of adjustment was improved by reducing the computational burden with presented simplified granularity adjustment and by introducing a sub-portfolio calculation with an upper limit for part of the sub--portfolios. Simplified granularity adjustment is used by at least some supervisors to assess Pillar 2 credit concentration risk.

The increased applicability of the measurements may be identified as one line of research. Juodis et al. (2009) assesses the practical calculations of concentration with the data of one bank, Bellalah et al. (2015) studied the sensitivity of granularity adjustment to risk indicators HHI and the Gini-index, as well as the sensitivity to credit portfolio risk parameters. Slime (2016) evaluates concentration risk add-ons. Concentration risk granularity add-ons are proposed by Lefcaditis, Tsamis and Leventides (2014) for Greek banks' portfolios while comparing corrections to a regulatory method, and Prorokowski, Prorokowski and Nteh (2019) study changes in regulatory methods for credit concentration risk. Research from Emmer and Tasche (2005) evaluate the effect of additional credit when the rest of the portfolio is diversified.

3. Methodology

Three parameters are the basis for analytical treatment of defaults in credit portfolios, and an estimation of these parameters is also essentially the IRB method in prudential regulation. It is assumed that credit has a certain probability of default (PD) and in the case of a credit default, a parameter loss given default (LGD) describes the percentage of loss. This loss is expressed as a ratio to the exposure amount of the credit at default, exposure at default (EAD). If each PD, LGD and EAD is identical for all exposures, a portfolio is homogenous. Maturity (M) of the exposure may also be treated as a risk parameter. If parameter values vary between exposures, the portfolio is heterogenous. In the time dimension, parameters can be constant or stochastic. Properties of portfolio losses can be described with expected loss (EL) and unexpected loss (UL). Within the Basel regulations' internal model regulation regime, risk parameters are linked with capital requirements if FI applies the IRB method in the calculation of capital requirements. In IRB method FI uses its own estimates at least on PD and in advanced methods also to EAD and LGD while EL and UL are calculated with the given formulas.

3.1. Asymptotic single risk factor framework

The Basel regulations' IRB formula is a combination of the Vasicek formula and Gordys result on conditional dependence (Hibbeln 2010). The ASRF formula's treatment of defaults has its foundation in the limit where the value of a firm's liabilities exceeds the value of the firm's assets. In the default model of Merton, the default of a firm occurs if the value of liabilities exceeds the value of the assets of the firm. Asset prices are drivers for defaults with given loan sizes and in the case of two firms, the dependence can be modelled through asset correlations. A further assumption is that the assets are correlated through one or several systematic risk factors. A well-known Black-Scholes formula for the valuation of asset pricing includes a stochastic evolvement of asset prices and sets the grounds for a valuation of the credit portfolio and for an analytical treatment of defaults. In the Black-Scholes formula the value of a firm's assets changes stochastically, and the variance of a normal distribution is a central parameter. The model is extensively applied to the valuation of stock prices, and the volatility of stock prices is then a factor that directly affects prices. A default occurs if the value of a firm's assets V_t at time t is below the debt of a firm B so that $V_t - B < 0$. The Basel 2 IRB formula is at a level of confidence as $\alpha = 0.999$.

$$UL_{ASRF} = \sum_{i=1}^{n} s_i \, \mathrm{E}\left(\widetilde{\mathrm{LGD}}_{\mathrm{i}} \,\middle|\, \tilde{x} = \Phi^{-1}(0.999) \right) \Phi\left(-\frac{\Phi^{-1}(\mathrm{PD}) + \sqrt{\rho}\,\Phi^{-1}(0.999)}{\sqrt{1-\rho}} \right) \tag{1}$$

IRB modifications to the formula, as explained by BCBS (2005b), are that EL is subtracted from (1), LGD is a downturn estimate, PD reflects a one-year default horizon and, correspondingly, a maturity correction is included. Also, a scaling factor of 1.06 is added to maintain an overall level of capital requirements as equal to the preceding Basel 1 regulation.

Each exposure class has its specific correlation formulas in the regulation. For large corporates and SMEs, the correlations depend on the company qualities within given limits. For SME exposures in retail, the correlation formula is set between 0.03 and 0.16, based on the exponential weighting of PD:

$$\rho = 0.03 \times \frac{1 - e^{-35 \times \mathrm{PD}}}{1 - e^{-35}} + 0.16 \times \left(1 - \frac{1 - e^{-35 \times \mathrm{PD}}}{1 - e^{-35}} \right) \tag{2}$$

The increase of PD decreases the correlation of PD, which reflects an assumption that with higher PD levels idiosyncratic risk is given more weight and systematic risk, dependence on other obligors, is given less weight. In the IRB formula there is an inverse linear function between asset correlations and PD, as a low PD implies high asset correlation. Multiplying $\mathrm{UL}_{\mathrm{ASRF}}$ by 1250 gives risk-weighted assets (RWA) of a counterparty or a portfolio. There are separate studies from an RWA viewpoint such as Baule and Tallau (2016) on the business cycle and cross-sectoral variation and Prorokowski (2017) on risk weight levels for sovereign credits.

The Monte-Carlo simulation is a somewhat burdensome but well-founded method for the assessment of loss and tail loss distributions of a credit portfolio with specified risk drivers and correlation structure between credits. These structures are present in the IRB model. The generation of idiosyncratic and systematic risk factors from standard normal distribution and comparison with the default threshold constructs a simulated loss distribution. The result on unexpected loss with Monte Carlo simulations ($\mathrm{UL}_{\mathrm{MC}}$) with VaR 99.9 % is portfolio-size structure-specific, and therefore informative also about credit and portfolio size effects. The ASRF formula, on the other hand, requires adjustment for the granular features of the portfolio.

3.2. Granularity adjustment

The development of granularity adjustment was initiated by Martin and Wilde (2002) and Gordy (2003), who developed analytical approximation for granularity as options for the burdensome of Monte Carlo simulations. This approach was essentially advanced by Gordy and Lutkebohmert (2013), who presented an improvement to the granularity adjustment and the application of adjustment was improved by reducing the computational burden with the presented simplified granularity adjustment and by introducing sub-portfolio calculation with an upper limit for part of the sub-portfolios.

The granularity adjustment of Gordy and Lutkebohmert (2013) uses portfolio exposure distribution and aligns the fully diversified IRB unexpected loss with non-diversified portfolios' corresponding unexpected loss. Using the portfolio IRB capital requirement K^* as in (1) based on unexpected loss and expected loss R_i for exposure I, a simplified granularity adjustment obtains the form

$$\mathrm{GA} = -\frac{1}{2K^*} \times \sum_{i=1}^{n} s_i^2 \; C_i \left(\delta \left(K_i + R_i \right) - K_i \right) \tag{3}$$

where δ is a regulatory parameter between 0 and 1 and

$$C_i = \frac{V\left[LGD_i\right] + E\left[LGD_i\right]^2}{E\left[LGD_i\right]}$$

We use in the text abbreviation GA for simplified granularity adjustment and the value 0.25 for δ following Gordy and Lutkebohmert (2013). Unexpected loss including ASRF and GA is $\mathrm{UL}_{\mathrm{ASRF+GA}}$. A common feature for the results of concentration risks of a portfolio is that changes in PD and LGD have high impacts on the results.

HHI is simply the sum of squares of exposure shares s_i

$$HHI = \sum_{i=1}^{n} s_i^2 \quad i = 1,\dots,n \tag{4}$$

Proportionality to HHI in GA means that with given homogenous portfolio parameters PD, LGD and M, the results can be obtained simply through the analysis of HHI. Based on previous research, the large credits have the potential to incorporate variation to HHI. The hypotheses for effects on large credits based on earlier research and chosen methodologies are as follows:

H_1: As the assumption of portfolio-invariance is relaxed, the effects of additional credits vary based on initial portfolio properties;

H_2: The effects from additional credits to the portfolio may vary between model-free HHI and model-based portfolio risk measurement;

H_3: The share of concentration risk in total credit risk is high at least for some credit portfolios when there are large credits.

All these hypotheses include an aspect that large credits do affect granularity concentration. When the three hypotheses are combined and tested, the effect of large credits is known from several directions of portfolio risk measurement.

4. Data

The introduction of one or several large credits has several impacts on the risk of a portfolio. The ASRF formula implies portfolio invariance, which means that additional exposure has the same effect regardless of which portfolio it is added to. This assumption may not be valid as relatively large credits are added to portfolios. Pillar 3 information on the portfolio is presented in Table 1 for one FI. We chose a total of 65 portfolios in institutions, central government and corporations as our grade level Pillar 3 data. The selection of FIs was from a list of European Central Bank-supervised financial entities in 2019, including 108 institutions. Although there are common guidelines BCBS (2015) for Pillar 3 reports, comparable data was available from a limited set of institutions and a minority of portfolios are subject to Pillar 3 reports as the IRB method is not used by all institutions, and even if an institution uses the method, some portfolios are outside the IRB method. One year of data is chosen as in one time point the reporting on Pillar 3 is based on the same regulation and the interest is in the portfolio structure instead of portfolio evolvement.

The literature on credit portfolios does give some guidance on how to construct realistic portfolios in terms of exposure distribution with a given portfolio size n. Heterogenous credit sizes are practically observed in research and Galaasen et al. (2020) presents an “80% to 20%” rule, stating that 20% of the largest credits constitute 80% of a portfolio’s exposure. Here portfolios are constructed using the FI’s public Pillar 3 reports and common assumptions of exact credit distributions. Using an approach similar to Slime (2016) and Krali and Gurov (2019), exposures may be assigned by using the assumption of log-normal distribution with

$$s_i = e^{z_i}$$

Conditioning with systematic risk X gives an expected value and variance

$$z_i \sim N\left(\mu'_A, \sigma'_A\right)$$

where

$$\mu'_A = E\left[s \middle| PD_i\right]$$

and

$$\sigma'_A = \sigma'_A\left[PD_i\right]$$

Additional assumptions on correlations are not applied. Slime (2016) uses log-normal (10.3) distribution in generating exposures.

Log-normal credit distributions are assigned by taking the published rating grade, notation R, level n_R, PD_R and LGD_R information. The two steps are as follows:

1. Generating n_R credits from log-normal distribution with μ'_A as average grade level EAD and with three assummed σ'_A levels 0.2 , 0.5 and 0.8. These will be referred to as small, medium and large variance accordingly in the text.

2. Scaling EAD_i are scaled to s_i summing to unity.

The sensitivity of variance choice is evaluated by alternative portfolio EAD assignment applied also by Gordy and Lutkebohmert (2013), assuming a different i^p with p = 0,…,10 is included. For $EAD_i = i^2$ portfolio with n exposures consists of exposures 1, 4, 9, 16,…, n^2.

With known grade level distributions, but without exact credit size information, there would be options either to assume that the portfolio has some large credits included in the portfolio with a given structure or alternatively that large credits could be assumed as an addition to the existing portfolio structure. We choose the latter for clarity, as we then separate the effects into two assumptions: the assumption of portfolio structure and the assumption on large credit size. The effect of the portfolio upper threshold is visualized in Figure 1 and Figure 2. Figure 1 presents the size distribution of a portfolio with initial n = 129 and after adding 30 large credits, total n = 159. μ'_A based on initial portfolio’s EAD average share, variance σ'_A is zero. One feature of the size distributions is that the lowest PD grade has the highest n combined with the highest average credit size. The added credits, presented by the green line, are the same size as other credits in the same grade.

Figure 2 presents the size distribution of a portfolio with a variance $\sigma'_A = 0.2$ and 30 large credits added. The size distribution now includes size variability within grade, and some credits in the second lowest PD grade are larger than the smallest in the lowest PD grade. The added credits, presented by a green line, are the same size as the largest credit in the portfolio.

The publications of Pillar 3 data are not in technically standard format, but the requirements on the level of data are standardized. FIs are obliged to follow the requirements on minimum information to be reported. The format of publication is not specified, and the requirements also leave room for interpretations. Some reports are in Excel and some are in written publication format. There are differences in the exposure classes shown, considering, for example, the retail SME part. The exact n or EAD by risk may not always be reported. The interpretation and use of rating grade data may require some supporting knowledge and information on prudential calculation to find comparable data points. There was, for example, a repeated error in table headings for one FI that without comparison to the FI's other prudential information would have affected the results in this research. The accuracy of numbers varies as, for example, FI reports PD as 0.0% for a specific grade and interpretations might vary from applying the minimum 0.03% in the applicable regulation or the rounding of PD with a maximum being 0.049%.

5. Results

Even before the addition of large credits, the initial portfolio structure varies considerably in the selected data. We first observe the results with a small subset of portfolios, which are identical in having the highest share of exposures and highest average exposure in the lowest PD grade and are therefore in the best credit quality class. The smallest 10 that filled the criteria were selected. These criteria resulted in portfolio sizes from 31 to 1481 counterparties. As noted in chapter 2, credit portfolio sizes below 4000 are especially candidates for remaining granularity. The whole set of the portfolios used consists of 65 portfolios, where the size varies between 27 to 3335 credits and the structure is not limited to those having a high share of exposure in the best risk grade. The assumption of the base portfolio credit composition is a first phase that incorporates large credits to a portfolio. In Table 2, data on portfolios show that variability assumptions are effective in producing different shares of the largest credit. Comparison of granularity measures with different variances, even without additional large credits, describes the effect of large exposures to portfolio risk measurement as the share of large credits is at least doubled in all portfolios.

A comparison to HHI from i^p allocation in Table 3 shows that with variance 0.2 the increase in exposure size corresponds to p = 1 allocation for nine out of then portfolios. With 0.5 the HHI is closer to p = 2 or p = 3 allocation and for 0.8 the correspondence is to larger p values. With the highest number of portfolios in the p = 2 allocation with initial n, the choices of log-normal variance present variability around HHI levels with quadratic p allocation.

As large credits are added, HHI decreases with the smallest variance for the nine smallest portfolios as seen in Table 4. The decrease is very strong for the smallest portfolios 1 and 2. When the initial portfolio size is 31 or 39 credits, the addition of 30 credits decreases HHI strongly. For these two smallest credits, the decrease becomes even stronger as variance is increased. With a larger variance HHI increases for portfolios 4–10 and the increase is at the highest above 50% in some portfolios.

A mixed evolvement of HHI by portfolios may be interpreted as an indication of varying concentration change depending on the initial portfolio features. For smaller portfolios the increase in portfolio size n contributes to a decrease of HHI, while for the larger ones the cumulation of large credits is reflected as an increase in this concentration indicator.

The increase in exposure amount is an intuitively clear part of portfolio risk development, as an increase of variance implies that larger credits are applied in terms of their relative size of portfolio. In selected portfolios exposure may multiply, or the increase may be limited to a small proportional change. The risk after changes in portfolio distribution are measured though the portfolio's average credit risk according to ASRF and through GA. As the share of the lowest PD grade increases, the average risk decreases. Still, variability in the evolvement of average risk is high.

GA is increasing overall through variance and decreasing through added credits for a portfolio. Portfolio 4 has a higher GA than portfolio 3, which is smaller. Portfolios 3 and 5 have fairly identical GA values. Portfolio 5 has the lowest UL% of all portfolios.

The share of granularity presented in Table 6 is higher overall for smaller portfolios, but changes in added credits decrease this share.

When granularity is described as a risk weight add-on multiplying GA by 1250, as in Table 7, the results have an increased comparability to the credit risk standardized method, not only the IRB method. It can also be stated that add-ons could be used to set the materiality threshold of a risk. Result 0 as a threshold that there wouldn't be an addition of risk weights due to concentration risk.

The GA method and MC simulation from 200,000 simulation runs are compared using the GA share of portfolio risk as in Table 9, to excess risk of MC simulations in comparison to ASRF. Both are expressed as a ratio to aggregate the portfolio risk of each method, and are aggregated for the horizons $t = 0, 5, 10, 15, 20, 25, 30$. Table 8 shows that the results are close for many portfolios, while for one portfolio, number 4, the difference is notably large. Portfolio 4 has the highest absolute GA of all portfolios and the highest UL% of portfolios where $n < 400$. In all cases, the share of granularity increases with variance.

For a small set of portfolios, evolvement in the share of granularity captured the concentration risk aspect well, and it was clear that the structure of the portfolio changed in various ways from the addition of new credits. We observe how this set evolves for a larger set of portfolios with different risk levels and with large sizes. Size classes are used to separate the results. The results are presented in Table 9. With the rating grade level low-variance allocation, the share of granularity decreases in size classes below 1000 and increases for the highest size class. With medium variability, the share of granularity increases for size classes above 1000. The increase in all classes is below 10%. With large variability, the share of granularity rises more than 20% in several size classes.

The risk weight add-on results are calculated from the average GA in Table 10. There is clear support that the smallest portfolios should have a granularity risk add-on attached to risk measurements. Also, for largest portfolios in the study, a threshold of 0 risk weight would leave these portfolios without a granularity add-on, unless there is a high number of large credits added to the portfolio. Even in the case of adding 50 large credits, a risk weight of 1 for the group of largest credits is low in relation to smaller portfolios.

6. Discussions and implications for future research

Results showed that a change in the portfolio structure through cumulated large credit affects the total portfolio exposure, the rate of unexpected loss in ASRF, and credit concentration risk as well. Heterogenous credit sizes were included through common assumptions of portfolio compositions, which imply a mix of various-sized credits, including some relatively small and some relatively large ones. The generation of a wider range of portfolios also enabled a broader view to the effects of concentrations. Credit portfolio risk assessment is affected by the size of its largest credits and we proceed to evaluate three hypotheses of chapter 4 based on the results.

6.1. Evaluation of the hypotheses

The measured concentration risk varies through three main characteristics: (1) the initial portfolio structure, (2) the number of large credits and (3) the size of large credits. These findings give support to hypothesis 1 on having varying results based on the initial portfolio in the absence of portfolio invariance. With smaller portfolios, the concentration's share of portfolio risk decreases, even when adding several large credits. While the HHI decreased strongly for smaller portfolios when adding large credits, the share of concentration risk diminished only modestly. For a potential difference between model-free and model-based measurement, hypothesis 2 was given support. For larger portfolios, the share of concentrations increases, especially as the size of larger credits is increased. Even for a set of portfolios with a similar structure of credits in the lowest risk grade, some change patterns in comparison to the initial portfolio risk were straightforward and some were more divided. The exposure amount in some cases increased very strongly. The average risk decreased in all cases. The concentration risk overall showed similar changes when measured with MC simulations and with GA. According to the results, granularity may present the largest share of a portfolio's total credit risk as the share of concentration through granularity for some portfolios exceeded the share of risk from ASRF portfolio risk measurement. The high share of concentration risk, in some cases over 80% of total credit risk, supported hypothesis 3 of having high concentration risk in some portfolios.

6.2. Implications for future research

Model based measurement of concentration risk requires considerations of the effects of large credits at least in the case of smaller portfolios. Both the count and size of large credits are relevant for granularity. The possibility of finding simple size-based rules is limited also by the effects of large credits on concentration risk. A portfolio exposure increase combined with an increase in the share of granularity within a portfolio is a combination that was observed for several portfolios. Accumulating large credits has effects through increasing the portfolio size, and through changing the size ratios among the credits.

The applicability of concentration risk measurements is extended by introducing a risk-weight add--on based on granularity measurement. Add-on has a unified prudential interpretation for institutions using standardized methodology and institutions using an IRB approach. The add-on also serves as a potential trigger for portfolio division between diversified and non-diversified portfolios and is additive as opposed to the relative trigger proposed by, for example, Gürtler, Heithecker and Hibbeln (2006). Comparisons to studies on risk weight may be extended. In comparison to the risk-weight level variations of Prorokowski (2017) for sovereigns, variation of risk weights through granularity seems to be important, at least when the granularity risk weight add-on is at least one RWA unit. This add-on may be further studied as a potential threshold between concentrated and non-concentrated portfolios.

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Appendix

Table 1

Pillar 3 information on institution portfolio for one IRB institutions

Grade	PD (%)	LGD (%)	n	EAD
1	0.100	14.4	98	3 753.0
2	0.200	7.5	10	228.0
3	0.400	33.8	7	19.0
4	0.600	34.8	8	14.0
5	0.900	57.0	3	13.0
6	6.700	51.7	3	27.0
Average	0.155	25.9		
Sum			**129**	**4 054.0**

Table 2

Size, portfolio risk and share of lowest PD grade in selected portfolios (%)

	Institution									
	1	2	3	4	5	6	7	8	9	10
n	31	39	129	196	230	274	391	426	431	1481
UL	0.18	0.28	0.61	0.75	0.12	0.57	0.61	0.68	1.01	1.01
RWA	2.2	3.6	7.7	9.4	1.5	7.1	7.6	8.5	12.6	12.7
n share of grade 1	80.65	79.49	75.97	77.55	90.87	76.28	69.82	73.24	78.89	78.87
EAD share of grade 1	99.00	92.75	92.58	99.96	99.76	98.04	90.05	96.11	96.20	91.78
Largest credit share (log-normal allocation)										
Variance = 0.2	4.0	3.0	0.9	0.7	0.5	0.5	0.3	0.3	0.3	0.1
Variance = 0.5	5.8	4.3	1.7	1.2	0.9	0.7	0.6	0.6	0.5	0.2
Variance = 0.8	9.6	7.0	3.6	2.8	2.0	1.2	1.2	1.5	1.1	0.4

Table 3

Correspondence between log-normal variance allocation and portfolio level i^p allocation for ten portfolios

Variance of log-normal distribution	Portfolio n	HHI corresponding to i^p with p less than						
		1	2	3	4	5	6	7
0.2	initial n, count of portfolios	9	1					
0.5			10					
0.8				7	3			
0.2	added 15 large credits, count of portfolios	9	1					
0.5			7	3	1			
0.8			1	3	2	2		2

Table 4

Change of HHI from initial portfolio (%)

Institution	1	2	3	4	5	6	7	8	9	10	Variance
Initial portfolio n	31	39	129	196	230	274	391	426	431	1481	
Added credits	HHI change from initial portfolio										
5	-16	-11	-4	-1	-1	-2	-1	-1	0	0	0.2
10	-28	-20	-7	-2	-3	-4	-2	-2	-1	0	
15	-37	-28	-10	-3	-4	-5	-3	-3	-1	0	
20	-44	-35	-14	-5	-6	-7	-4	-5	-2	0	
25	-50	-40	-17	-7	-7	-9	-5	-6	-2	1	
30	-54	-45	-17	-11	-10	-5	-6	-7	-3	1	
5	-10	-4	2	3	1	1	9	7	5	2	0.5
10	-23	-15	2	3	1	2	16	12	9	4	
15	-34	-25	-1	3	1	1	20	16	12	6	
20	-43	-34	-4	1	0	1	22	18	14	8	
25	-50	-41	-7	-1	-1	-1	24	19	15	9	
30	-55	-47	-8	-4	38	-3	24	19	15	11	
5	-10	-7	11	11	8	14	34	27	21	10	0.8
10	-28	-23	12	14	11	20	51	40	33	18	
15	-41	-36	8	12	11	21	58	46	40	25	
20	-50	-46	3	8	10	21	60	48	43	31	
25	-57	-53	-3	4	8	19	59	47	44	37	
30	-62	-58	-3	-3	84	-1	56	45	44	41	

Table 5
Granularity adjustment (GA) for portfolio with varying variance

Institution	1	2	3	4	5	6	7	8	9	10	Variance
Initial portfolio n	31	39	129	196	230	274	391	426	431	1481	
Added credits	Granularity adjustment										
0	0.0069	0.0097	0.0030	0.0031	0.0029	0.0018	0.0016	0.0013	0.0016	0.0004	0.2
5	0.0061	0.0080	0.0029	0.0031	0.0029	0.0017	0.0016	0.0013	0.0016	0.0004	
10	0.0055	0.0068	0.0028	0.0031	0.0029	0.0017	0.0016	0.0013	0.0016	0.0004	
15	0.0050	0.0059	0.0027	0.0030	0.0028	0.0017	0.0016	0.0013	0.0016	0.0004	
20	0.0045	0.0052	0.0027	0.0030	0.0028	0.0017	0.0016	0.0013	0.0016	0.0004	
25	0.0041	0.0047	0.0026	0.0029	0.0028	0.0016	0.0016	0.0013	0.0016	0.0004	
30	0.0038	0.0043	0.0026	0.0028	0.0027	0.0017	0.0016	0.0012	0.0016	0.0004	
0	0.0080	0.0119	0.0036	0.0038	0.0034	0.0021	0.0020	0.0016	0.0019	0.0004	0.5
5	0.0079	0.0092	0.0037	0.0039	0.0035	0.0021	0.0022	0.0018	0.0020	0.0004	
10	0.0071	0.0076	0.0036	0.0039	0.0035	0.0021	0.0024	0.0018	0.0021	0.0004	
15	0.0063	0.0065	0.0036	0.0039	0.0036	0.0021	0.0024	0.0019	0.0022	0.0004	
20	0.0057	0.0057	0.0035	0.0038	0.0035	0.0021	0.0025	0.0019	0.0022	0.0004	
25	0.0051	0.0050	0.0034	0.0038	0.0035	0.0021	0.0025	0.0020	0.0022	0.0005	
30	0.0047	0.0045	0.0033	0.0037	0.0037	0.0021	0.0026	0.0020	0.0023	0.0005	
0	0.0106	0.0156	0.0047	0.0054	0.0047	0.0029	0.0030	0.0025	0.0028	0.0006	0.8
5	0.0109	0.0116	0.0052	0.0060	0.0051	0.0033	0.0040	0.0031	0.0033	0.0006	
10	0.0094	0.0092	0.0052	0.0061	0.0053	0.0035	0.0045	0.0035	0.0037	0.0007	
15	0.0081	0.0077	0.0051	0.0060	0.0054	0.0036	0.0048	0.0036	0.0039	0.0007	
20	0.0070	0.0066	0.0049	0.0058	0.0054	0.0036	0.0049	0.0037	0.0040	0.0007	
25	0.0062	0.0058	0.0047	0.0056	0.0053	0.0035	0.0049	0.0037	0.0040	0.0008	
30	0.0055	0.0051	0.0047	0.0052	0.0057	0.0030	0.0049	0.0036	0.0040	0.0008	

Table 6
Granularity adjustments share of portfolio risk UL = UL_{ASRF} + GA (%)

Institution	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**Variance**
Initial portfolio n	**31**	**39**	**129**	**196**	**230**	**274**	**391**	**426**	**431**	**1481**	
Added credits	**GA share of portfolio risk**										
0	80	77	33	29	71	24	21	16	14	3	0.2
5	79	76	33	29	71	23	21	16	14	3	
10	78	74	32	29	71	23	21	16	14	3	
15	77	72	32	29	71	23	21	16	14	3	
20	76	71	31	28	71	23	21	16	14	3	
25	75	69	30	28	71	23	21	16	14	3	
30	74	68	31	27	70	23	21	16	14	3	
0	82	81	37	34	75	27	25	19	16	4	0.5
5	84	79	38	34	75	27	27	21	17	4	
10	83	78	38	34	75	28	28	22	18	4	
15	83	76	38	34	76	28	29	22	18	4	
20	82	75	37	34	76	28	30	23	18	4	
25	80	73	37	33	76	28	31	23	19	4	
30	79	72	37	33	82	27	31	23	19	4	
0	86	85	43	42	80	33	33	27	21	6	0.8
5	88	84	46	44	82	37	40	32	25	6	
10	88	82	47	45	82	38	44	34	27	6	
15	87	81	47	44	83	39	46	36	28	7	
20	85	79	46	44	83	39	46	36	29	7	
25	84	77	45	43	83	39	47	36	29	7	
30	83	76	46	41	84	35	47	36	29	4	

Table 7
Granularity adjustment formulated a as risk weight add-on, 1250 × GA

Portfolio risk weight add-ons with 1 to 30 added credits										
Portfolio	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
Variance										
0.2	5–9	5–12	3–4	3–4	3–4	2	2	2	2	0
0.5	6–10	6–15	4–5	5	4–5	3	3	2	3	1
0.8	7–13	6–19	6–7	7–8	6–7	4	4–6	3–5	3–5	1

Table 8
Granularity adjustments share of portfolio risk (%)

Institution	1	2	3	4	5	6	7	8	9	10	Variance
Var 99.9	84	68	30	67	72	29	18	21	14	2	0.2
	84	74	37	70	79	36	27	32	18	4	0.5
	81	74	47	76	83	50	42	51	31	8	0.8
GA	77	72	32	28	71	23	21	16	14	3	0.2
	82	76	37	34	76	27	29	22	18	4	0.5
	86	80	46	43	83	37	43	34	27	6	0.8

Note: MC simulations described in chapter 3 is UL_{MC} and for GA share of $UL_{ASRF+GA}$.

Table 9
Relative change in the ratio of granularity's share of portfolio UL compared with initial portfolio

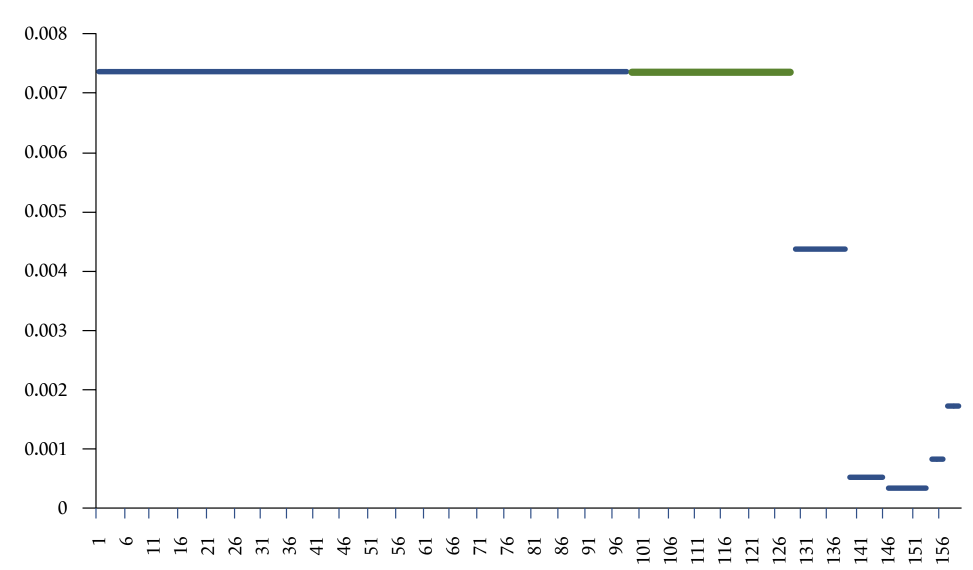
Portfolio n size class	Number of portfolios	Added credits						Variance log-normal allocation
		+1	+10	+20	+30	+40	+50	
< 100	10	0.981 (0.019)	0.836 (0.135)	0.732 (0.181)	0.657 (0.199)	0.599 (0.207)	0.552 (0.209)	0.2
100–250	11	1.002 (0.028)	0.968 (0.14)	0.909 (0.032)	0.852 (0.043)	0.802 (0.052)	0.757 (0.058)	
250–500	10	0.998 (0.002)	0.984 (0.018)	0.967 (0.032)	0.949 (0.043)	0.931 (0.052)	0.914 (0.058)	
500–1000	14	0.999 (0)	0.988 (0.005)	0.976 (0.01)	0.964 (0.015)	0.952 (0.021)	0.941 (0.026)	
1000–2000	9	1.000 (0.003)	1.000 (0.021)	0.994 (0.023)	0.986 (0.018)	0.977 (0.011)	0.967 (0.01)	
2000–3500	11	1.004 (0.014)	1.030 (0.097)	1.043 (0.137)	1.048 (0.15)	1.048 (0.15)	1.046 (0.143)	
< 100	10	0.985 (0.021)	0.854 (0.146)	0.751 (0.193)	0.674 (0.212)	0.614 (0.218)	0.565 (0.219)	0.5
100–250	11	1.011 (0.03)	1.012 (0.159)	0.959 (0.085)	0.901 (0.105)	0.848 (0.117)	0.800 (0.124)	
250–500	10	1.002 (0.007)	1.012 (0.053)	1.013 (0.085)	1.007 (0.105)	0.997 (0.117)	0.984 (0.124)	
500–1000	14	1.000 (0.001)	1.003 (0.008)	1.004 (0.015)	1.002 (0.022)	0.999 (0.029)	0.994 (0.035)	
1000–2000	9	1.003 (0.008)	1.018 (0.047)	1.021 (0.053)	1.018 (0.045)	1.013 (0.032)	1.008 (0.019)	
2000–3500	11	1.005 (0.013)	1.040 (0.088)	1.063 (0.123)	1.077 (0.134)	1.087 (0.133)	1.094 (0.127)	
< 100	10	0.996 (0.034)	0.875 (0.169)	0.762 (0.212)	0.678 (0.226)	0.613 (0.23)	0.560 (0.229)	0.8
100–250	11	1.030 (0.038)	1.081 (0.184)	1.026 (0.196)	0.959 (0.216)	0.894 (0.22)	0.836 (0.217)	
250–500	10	1.016 (0.021)	1.110 (0.14)	1.153 (0.196)	1.164 (0.216)	1.157 (0.22)	1.141 (0.217)	
500–1000	14	1.011 (0.009)	1.089 (0.068)	1.147 (0.108)	1.184 (0.133)	1.207 (0.149)	1.220 (0.159)	
1000–2000	9	1.011 (0.011)	1.080 (0.05)	1.127 (0.047)	1.159 (0.05)	1.184 (0.069)	1.202 (0.093)	
2000–3500	11	1.010 (0.008)	1.085 (0.056)	1.151 (0.083)	1.205 (0.104)	1.251 (0.124)	1.291 (0.145)	

Note: variance of ratio in parentheses.

Table 10
Granularity adjustment formulated a as risk weight add-on

Granularity as a risk-weight add-on								
Portfolio n size class	Initial portfolio	Added credits						Variance log-normal allocation
		+1	+10	+20	+30	+40	+50	
< 100	23	23	18	14	11	9	8	0.2
100–250	7	7	8	8	7	6	5	
250–500	4	4	4	4	4	4	4	
500–1000	2	2	2	2	2	2	2	
1000–2000	1	1	1	1	1	1	1	
2000–3500	0	0	0	0	0	1	1	

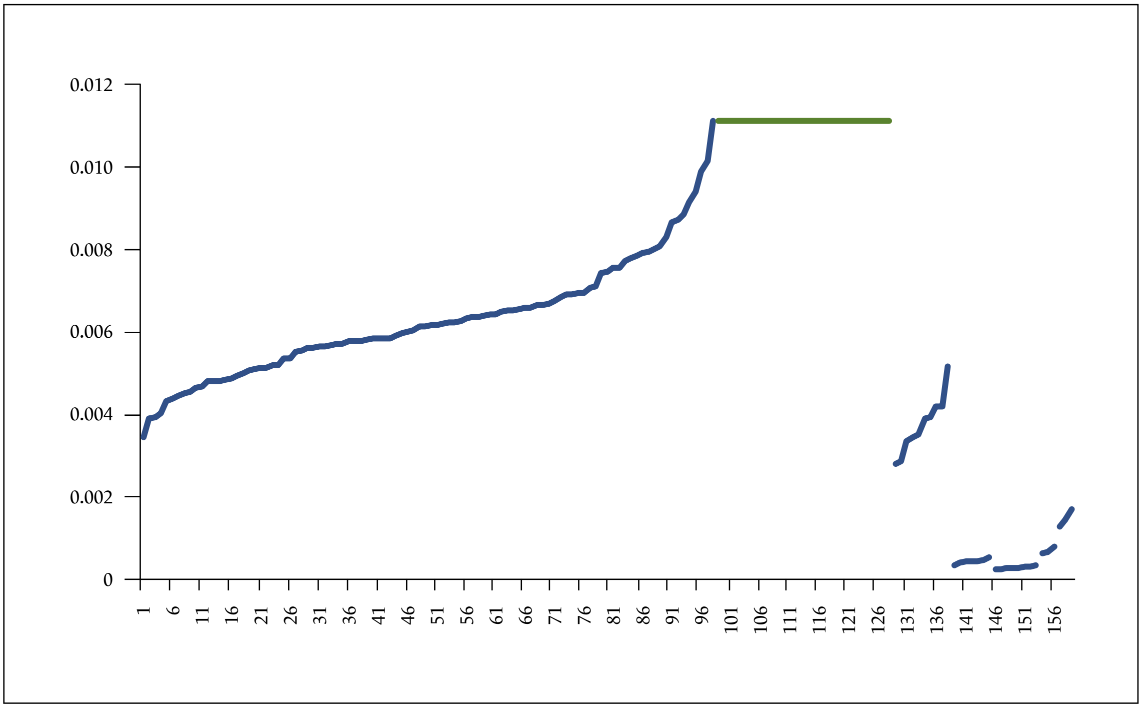
Figure 1
Size distribution of portfolio 3 with variance = 0.0, initial n = 129 and added 30 credits to grade 1



Note: total n = 159. Ordered by PD at grade level from lowest to highest. Green line presents added credits.

Figure 2

Size distribution of portfolio 3 with variance = 0.2, initial n = 129 and added 30 credits to grade 1



Note: total n = 159. Ordered primarily by PD-grade from lowest to highest and secondarily by size within grade. Green line presents added credits.

Czy duże ekspozycje kredytowe są źródłem ryzyka koncentracji?

Streszczenie

Jaki wpływ mają duże, indywidualne kredyty na ryzyko kredytowe banku? Ponieważ duże kredyty w istotny sposób wpływają na stopień granularności portfela kredytowego, powinny być przedmiotem szczególnego zainteresowania analityków. Należy podkreślić, że w ostatnich latach opracowano wiele metod analitycznych i symulacyjnych umożliwiających pomiar granularności portfeli kredytowych. Wpływ dużych kredytów na ryzyko portfeli nabiera szczególnego znaczenia w odniesieniu do regulowanych instytucji finansowych. Istnienie regulacji bankowych wpływa na zakres badań dotyczących ryzyka koncentracji portfela kredytowego, bo instytucje udzielające kredytów są w większości instytucjami regulowanymi, a nawet jeśli nie podlegają regulacjom, to należy się spodziewać, że będą stosować znane metody pomiaru ryzyka portfelowego.

Punktem wyjścia pomiaru ryzyka kredytowego przyjętym w artykule jest podejście oparte na asymptotycznym modelu jednoczynnikowym (*asymptotic single risk factor* – ASRF), uwzględniające pomiar wartości narażonej na ryzyko (*Value at Risk*) dla prawdopodobieństwa 99,9%. Metoda ta jest podstawą pomiaru ryzyka w 1. filarze regulacji bazylejskiej (Bazylea II) dla instytucji stosujących metodę ratingów wewnętrznych (IRB – *internal ratings-based approach*). Z kolei w filarze 2. tych regulacji wymaga się wprowadzenia ilościowych metod pomiaru ryzyka koncentracji. Badania w tym zakresie rozwinęły się w kierunku wprowadzania korekt wyznaczanych miar ryzyka kredytowego w celu uwzględnienia wpływu stopnia granularności portfela na łączne ryzyko kredytowe. Źródłem danych wykorzystanych w badaniu są sprawozdania instytucji finansowych dotyczące ekspozycji należących do poszczególnych klas ryzyka kredytowego (*rating grade level data*). Publikowanie tych informacji jest obligatoryjne, zgodnie z filarem 3. regulacji bazylejskiej. Raporty instytucji finansowych, które początkowo obejmowały od 27 do 14 mln kredytów i dotyczyły portfeli liczących mniej niż 3500 kredytów, zawierały wyniki wcześniejszych badań odnoszących się do zagadnienia granularności. Uzyskane rezultaty potwierdziły wcześniejsze wyniki badań i wskazały na nowe kryterium uzyskania progu dla zbliżonych zakresów wielkości kredytów, przy czym na próg ten wpływają duże indywidualne ekspozycje kredytowe. Tym nowym kryterium jest miara wag ryzyka lub korekta zwiększająca wagi ryzyka z tytułu ryzyka koncentracji. Dodatkową zaletą metody podwyższonej wagi ryzyka jest to, że może być stosowana przez banki wykorzystujące metodę wewnętrznych ratingów (IRB) oraz banki stosujące metodę standardową w zakresie wyznaczania wymogu kapitałowego z tytułu ryzyka kredytowego. Uzyskane wyniki mogą mieć również zastosowanie w przypadku wprowadzenia reform regulacyjnych Bazylea III z *input floors*, czyli dolnymi progami wag ryzyka, nakładającymi ograniczenia na wyniki metody IRB.

Wyniki analizy wskazują, że kumulacja dużych kredytów o jednakowej wielkości w jednym portfelu zawiera zarówno rosnący, jak i malejący komponent ryzyka koncentracji dla tego portfela. W portfelach liczących najmniej kredytów czynnik granularności odpowiada za 80% ryzyka kredytowego mierzonego wartością narażoną na ryzyko (*Value-at-Risk*). Zastosowanie formuły podwyższonej wagi ryzyka (*risk-weight add-on*) przez instytucje wykorzystujące standardowe metody oceny ryzyka oraz instytucje bazujące na IRB wiąże się z podwyższeniem wagi ryzyka dla tych instytucji o ponad 20% w przypadku portfeli liczących mniej niż 100 kredytów. Jest to interesujący rezultat dla analizy ryzyka w małych portfelach zawierających ekspozycje rządowe (*sovereign*), dla których waga ryzyka wynosi 0%,

lub w portfelach z ekspozycjami na instytucje o najwyższej wiarygodności kredytowej, gdy waga ryzyka wynosi 20%. W przypadku portfeli zawierających od 2000 do 3500 kredytów nie przewiduje się konieczności dodatkowego podwyższenia wagi ryzyka. Jeśli jednak dojdzie do kumulacji 50 dużych kredytów, to dodatkowo waga ryzyka zwiększa się o 1%.

Badanie dotyczy przede wszystkim wielkości dużych kredytów oraz funkcji rozkładu kredytów w portfelu. Średnia wielkość kredytu w każdej klasie ratingowej i w każdym portfelu jest znana, natomiast dokonano analizy wrażliwości wyników na odchylenia wybranych wartości od założeń przyjętych w głównej części badania.

Słowa kluczowe: pomiar ryzyka, ryzyko koncentracji, ryzyko kredytowe, regulacje bazylejskie