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Which hallmarks of optimal monetary policy rules matter in Poland? A stochastic dominance approach

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Abstract

We employ a stochastic dominance approach to find and rank alternative optimal simple monetary rules in the rational expectations model of the Polish economy, using a new algorithm to calculate the distributions of the optimal central bank welfare loss and policy feedback parameters. We apply this framework to the Erceg, Henderson and Levin (2000) model estimated for the Polish economy using quarterly data for 1995–2021 and examine monetary policy rule hallmarks for the welfare--loss-minimising central bank. We confirm the importance of policymakers' interest rate smoothing incentives and introducing variables of real economic activity and wages into optimal monetary policy rules. The central bank's choice of response variables depends on the dynamic specification of the policy rule and the interest rate smoothing mechanism. With an interest rate smoothing mechanism, the contemporaneous monetary policy rule that reacts to inflation, real wages, and the output gap, minimises the welfare loss for all decision makers admitting first-degree stochastic dominance preferences.

Keywords: stochastic dominance, optimal monetary policy, welfare loss distribution, DSGE model, parameter uncertainty

JEL: E47, E52, E61

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1. Introduction

A generally accepted principle of economics states that policymakers in central banks should act optimally (see Tinbergen 1952; Blinder 1999; Clarida, Galí, Gertler 1999; Taylor 1999; Woodford 2003). When making policy decisions, agents face the problem of constrained optimisation. The goal of an optimal central bank is to choose a policy instrument to minimise the expected welfare loss subject to dynamic macroeconomic equations that contain forward-looking rational expectations. Several sources of uncertainty can disturb the monetary policy rules (see Poole 1998; Blinder 1999; Goodhart 1999; Onatski, Williams 2003; Woodford 2003; Greenspan 2004). The primary source of this randomness is exogenous shocks disturbing the macroeconomic variables from their steady-state values. The Bayesian approach to macroeconomic modelling assumes that the posterior distribution of model parameters is another source of uncertainty. Many researchers and central bank practitioners emphasise that, due to uncertainty, a little stodginess on the part of the central bank policymakers is entirely appropriate (see Blinder 1999). Moreover, Chow (1975) reports no explicit dependence between the parameter uncertainty and the policy rules. Thus, quantitative research on the impact of economic uncertainty on the shape of optimal macroeconomic policies is required.

The usual approach to optimal monetary policy design in dynamic stochastic general equilibrium (DSGE) models assumes that the fixed values of structural model parameters are known with certainty (see Erceg, Henderson, Levin 2000; Giannoni, Woodford 2002; Taylor, Williams 2010 and references therein).[1] When min-max robust policy rules are used, the distributions of parameters are usually not available. In the first step, policymakers consider the worst-case scenario by maximising the welfare loss over the range of plausible parameter values or models; then, in the second stage, they minimise this worst-case value of welfare loss with respect to policy instruments (cf. Onatski, Williams 2003; Kendrick 2005; Hansen, Sargent 2010). In several research papers, the authors, using the min-max technique, prove that the robust optimal policy rule causes more aggressive responses of the interest rate to inflation and output gap shocks than in the case of parameter certainty (cf. Onatski, Stock 2002; Giannoni 2002, 2007). It is also possible to construct a mean robust monetary policy rule for models with parameter uncertainty (see Justiniano, Preston 2010; Górajski 2018). Using this approach, we minimise the expected value of welfare loss, in which the expectation is also taken with respect to the random model's parameters. In these approaches, both robust policy reaction functions are derived with certainty. Moreover, robust Bayesian rules are designed to account for model and parameter uncertainty (see Cogley et al. 2011; Levine, McAdam, Pearlman 2012 and references therein). In all of the above approaches, we observe inconsistency between the parameters of optimal or robust policy rules and the macroeconomic model. Policymakers treat optimal or robust policy feedback coefficients as fixed unique numbers, even though the structural non-policy parameters are random outcomes from the posterior distributions as the assumed macroeconomic model is observed with parameter uncertainty. Instead, we assume that the optimal policy coefficients are random variables with probability distributions inherited from the posterior distributions of the structural model parameters to resolve this inconsistency.

[1] Assuming certainty about the economic model, Milo et al. (2013), Bogusz, Górajski and Ulrichs (2015) and Górajski and Ulrichs (2016) present an analysis of optimal and risk-sensitive interest rate rules in vector autoregressive models for the Polish economy.

In this paper, we add to the existing knowledge by considering the problem of ranking a broad set of simple monetary policy rules within a DSGE model of the Polish economy. We consider a Bayesian policymaker with no predetermined attitude towards welfare loss who assesses the effectiveness of policy actions by comparing welfare loss distributions using k-degree stochastic dominance (SDk) (see Hadar, Russell 1969). Stochastic dominance is often used for social welfare comparisons (see Deaton 1997). We apply the algorithm of Górajski and Kuchta (2021) to calculate the distributions of minimised welfare losses and find SDk-optimal policy rules. We recall that these rules are robust to the whole uncertainty about the structure of the economic model. The SDk relation $L_1 \leq_{SDk} L_2$ between two non-negative welfare losses with bounded support $(0, L^*)$ assumes that the Bayesian policymaker compares infinitely many expected disutilities

$$\int_0^{L^*} u(x)\, dF_{L_1}(x) \leq \int_0^{L^*} u(x)\, dF_{L_2}(x) \quad (1)$$

for all disutility functions u such that their derivatives u' are absolutely monotonic[2] with strict inequality for some u, and where F_{L_1}, F_{L_2} are cumulated distribution functions of L_1 and L_2. We recall that the SD1 policymaker's attitude towards welfare losses assumes that (1) holds for all non-decreasing disutility functions, whereas SD2's preferences require the policymaker to be risk averse regarding welfare losses and restrict the disutility functions to non-decreasing and convex ones.

Within this framework, we answer a question that many researchers have asked: which hallmarks of monetary policy rules matter for the welfare-maximising central bank in Poland? The following hypotheses are the specifications of the above-mentioned research question. First, monetary policy rules that react only to inflation are suboptimal. Second, policy rules that respond only to inflation and the output gap do not necessarily limit welfare losses more than policy rules that react only to inflation and real wages. Third, adding interest rate smoothing to policy rules decreases the welfare loss distributions. We verify the above hypotheses by investigating the simple and implementable monetary policy rules, in the sense of Schmitt-Grohe and Uribe (2007), under a welfare-loss-minimising criterion.[3] We use Erceg, Henderson and Levin's (2000) small-scale closed-economy model estimated for the Polish economy.[4] Our model is one of the simplest models for which the divine coincidence in the spirit of Blanchard and Gali (2007) does not hold. Consequently, the solution for an optimal monetary policy depends on the values of the underlying parameters. Although Poland is an example of a small-scale open economy, we focus on the closed-economy model. This simplification is motivated by the results obtained by Gali and Monacelli (2005) and Justiniano and Preston (2010) for small open economies. The latter paper shows that optimal policies under parameter uncertainty exhibit a lack of exchange rate response in the estimated models of three small open economies. Gali and Monacelli (2005) prove that the welfare loss function for a small open economy does not include foreign variables if the specific calibration is imposed on the open-economy settings. It is worth noting that Baranowski and Kuchta (2014), Krajewski (2015) and Baranowski et al. (2016) estimate or calibrate the closed-economy DSGE

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2 A function $f:(0,L^*) \to (0, \infty)$ is absolutely monotonic if it has derivatives of all orders and $f^{(k)}(x) \geq 0$ for all $x \in (0, L^*)$ and $k = 1, 2, \ldots$

3 Erceg, Henderson and Levin (2000) show that the rule that solves the Ramsey problem in their model contains the non- -observed components, like technological shocks. Thus, it is not implementable. On the contrary, Williams (2003) reveals that, even in a richly specified DSGE model, a simple rule can perform quite well compared with a fully optimal policy rule that solves the Ramsey problem.

4 Brzoza-Brzezina and Suda (2021) emphasize that DSGE models are an appropriate tool for comparing alternative policy rules.

model for Poland. In a recent work, Cieślik and Teresiński (2020) also analyse the medium-scale closed--economy DSGE model for the Polish economy.

We consider backward-looking, contemporaneous and forward-looking monetary policy rules. We examine several dimensions of implementable monetary policy rules for a welfare-loss-minimising central bank. We confirm the importance of introducing real variables into the monetary policy rules. Moreover, the central bank's choice of response variables depends on the dynamic specification of the rules. For a broad set of monetary policy rules, we show that the fully specified contemporaneous monetary policy rule with an interest rate smoothing mechanism minimises the welfare loss distribution with respect to the first-degree stochastic dominance ordering.

The paper is organised as follows. The following section briefly presents the log-linear equations of the Erceg, Henderson and Levin (2000) model and provides the Bayesian estimation procedure. Section 3 contains the novel algorithm for measuring the uncertainty of optimal policy rules and sets out the decision framework. Section 4 presents our empirical results of optimal monetary policy rules for the Polish economy. Section 5 concludes the paper.

2. The theoretical model

This section briefly describes the new Keynesian model proposed by Erceg, Henderson and Levin (2000) with sticky prices and wages à la Calvo (1983). Our economy is populated by final-good firms, intermediate-good firms, labour agencies and households. Households consume, save and provide labour agencies with differentiated labour services. Households are subject to two structural disturbances: preference shocks and labour supply shocks. The imperfect substitution of labour allows households to influence their nominal wages, which are set according to the Calvo scheme. A labour agency rents the labour services on the perfectly competitive market to intermediate-good firms. Intermediate-good firms produce differentiated intermediate goods using constant returns to scale technology. Labour productivity is driven by technology shocks. The final-good producer buys the intermediate goods on a monopolistically competitive market where the prices are set according to the Calvo scheme. The final-good firms use them to produce the final goods and sell them to households.[5]

Below we present the log-linear form of the theoretical model.[6]

$$\hat{y}_t = E_t\left\{\hat{y}_{t+1}\right\} - \frac{1}{\delta_c} E_t\left\{\hat{r}_t - \hat{\pi}_{t+1} + \hat{\varepsilon}^b_{t+1} - \hat{\varepsilon}^b_t\right\} \tag{2}$$

$$\hat{\pi}_t = \beta E_t\left\{\hat{\pi}_{t+1}\right\} + \frac{\left(1-\theta_p\right)\left(1-\beta\theta_p\right)}{\theta_p}\widehat{rmc}_t \tag{3}$$

$$\widehat{rmc}_t = \hat{w}_t - \hat{\varepsilon}^a_t \tag{4}$$

$$\hat{w}_t = \frac{\left(1-\theta_w\right)\left(1-\beta\theta_w\right)}{\theta_w}\frac{b_w\tau_w}{\delta_l\left(1+\tau_w\right)+\tau_w}\widehat{mrs}_t + b_w\beta E_t\left\{\hat{\pi}_{t+1} + \hat{w}_{t+1}\right\} - b_w\hat{\pi}_t + b_w\hat{w}_{t-1} \tag{5}$$

[5] A detailed description of the theoretical model is provided in Appendix A.

[6] All the variables are expressed as the percentage deviation from the steady state.

$$\widehat{mrs}_t = \hat{\varepsilon}_t^l + (\delta_l + \delta_c)\,\hat{y}_t - \delta_l \hat{\varepsilon}_t^a \tag{6}$$

$$\hat{\varepsilon}_t^a = \rho_a \hat{\varepsilon}_{t-1}^a + \sigma_a\,\eta_t^a;\eta_t^a \sim i.i.d.\,N(0,1) \tag{7}$$

$$\hat{\varepsilon}_t^b = \rho_b \hat{\varepsilon}_{t-1}^b + \sigma_b\,\eta_t^b;\eta_t^b \sim i.i.d.\,N(0,1) \tag{8}$$

$$\hat{\varepsilon}_t^l = \rho_l \hat{\varepsilon}_{t-1}^l + \sigma_l\,\eta_t^l;\eta_t^l \sim i.i.d.\,N(0,1) \tag{9}$$

where: $\hat{y}_t$ is the output gap, $\hat{r}_t$ is the nominal interest rate (policy instrument), $\hat{\pi}_t$ is the inflation rate, $\widehat{rmc}_t$ is the real marginal cost, $\hat{w}_t$ is the real wage, $\widehat{mrs}_t$ is the marginal rate of substitution, $\hat{\varepsilon}_t^a$, $\hat{\varepsilon}_t^b$ and $\hat{\varepsilon}_t^l$ are shocks to the technology, preferences and labour supply, $b_w = \dfrac{\theta_w\left(\delta_l\left(1+\tau_w\right)+\tau_w\right)}{\delta_l\left(1+\tau_w\right)+\tau_w-\left(1-\beta\theta_w\right)\left(1-\theta_w\right)\delta_l\left(1+\tau_w\right)+\beta\theta_w^2\left(\delta_l\left(1+\tau_w\right)+\tau_w\right)} > 0$, $\delta_c > 0$ is the relative risk aversion parameter, $\beta \in (0;1)$ is the discount factor, $\theta_p \in (0;1)$ is the price stickiness parameter, $\theta_w \in (0;1)$ is the wage stickiness parameter, $\delta_l > 0$ is the inverse of the Frisch labour elasticity, $\tau_w > 0$ is the wage markup, ρ_a, ρ_b, $\rho_l \in (0;1)$ are the autoregressive parameters for a technology shock, preference shock and labour supply shock, σ_a, $\sigma_b > 0$ and $\sigma_l > 0$ are the standard deviation of technology innovation η_t^a, preference shock innovation η_t^b and labour supply innovation η_t^l. In the above-mentioned linear rational expectations model, eq. (2) represents the dynamic IS curve, eq. (3) is the Phillips curve for inflation, eq. (4) defines the real marginal cost, eq. (5) is the Phillips curve for real wages, eq. (6) defines the marginal rate of substitution and eqs (7)–(9) define the stochastic processes for structural shocks.

2.1. Alternative monetary policy rules

The DSGE model, eqs (2)–(9), should be closed by a policy rule setting the interest rate level. Traditionally, the new Keynesian DSGE models use a Taylor-type rule that links the interest rate with the endogenous variables. Although the original Taylor (1993) rule sets the federal fund rate as a function of the inflation rate over the previous year and the output gap, it has been extensively modified, obtaining a vast number of different forms. We limit ourselves to a set of implementable monetary policy rules in the sense of Schmitt-Grohe and Uribe (2007). Hence, the set of interest rate rules applies the following general specification of the Taylor-type rule, including only measurable variables from the theoretical model:

$$\hat{r}_t = \rho_r \hat{r}_{t-1} + (1+\phi_\pi)\,E_t\{\hat{\pi}_{t+i}\} + \phi_y E_t\{\hat{y}_{t+i}\} + \phi_w E_t\{\hat{w}_{t+i}\} + \hat{\varepsilon}_t^r \tag{10}$$

where $i \in \{-1;0;1\}$ denote the dynamic specification of a policy rule as backward-looking ($i = -1$), contemporaneous ($i = 0$) or forward-looking ($i = 1$), $\hat{\varepsilon}_t^r = \sigma_r \eta_t^r$, $\eta_t^r \sim i.i.d.\,N(0,1)$ is a monetary policy shock, ρ_r is the interest rate smoothing parameter and ϕ_π, ϕ_y and ϕ_w are the inflation, output and real wage reaction parameters, respectively.

The interest rate rule (10) includes interest rate smoothing and reactions to inflation, output and the real wage. Although the first three components are fairly standard and well motivated, the last one may raise certain doubts. We include the real wage to emphasise the role of an alternative measure of economic activity and a variable that links more directly to the labour market.[7] Moreover, we allow for 0th restrictions on parameters ρ, ϕ_π, ϕ_y, ϕ_w.

As a result, we obtain eight different specifications of the Taylor rule (see Table 1). Hence, we work with 24 new Keynesian models differing by a monetary policy rule.[8] We use the letter l to denote both a policy rule from Table 1 and the DSGE model, eqs (2)–(9), with the policy rule specification l given by eq. (10) with some 0th restrictions on policy parameters.

Table 1

Specifications of simple monetary policy rules

Rule number	Functional form
1	$\hat{r}_t = (1+\phi_\pi) E_t\{\hat{\pi}_{t+i}\} + \hat{\varepsilon}_t^r$
2	$\hat{r}_t = (1+\phi_\pi) E_t\{\hat{\pi}_{t+i}\} + \phi_w E_t\{\hat{w}_{t+i}\} + \hat{\varepsilon}_t^r$
3	$\hat{r}_t = (1+\phi_\pi) E_t\{\hat{\pi}_{t+i}\} + \phi_y E_t\{\hat{y}_{t+i}\} + \hat{\varepsilon}_t^r$
4	$\hat{r}_t = (1+\phi_\pi) E_t\{\hat{\pi}_{t+i}\} + \phi_y E_t\{\hat{y}_{t+i}\} + \phi_w E_t\{\hat{w}_{t+i}\} + \hat{\varepsilon}_t^r$
5	$\hat{r}_t = \rho_r \hat{r}_{t-1} + (1+\phi_\pi) E_t\{\hat{\pi}_{t+i}\} + \hat{\varepsilon}_t^r$
6	$\hat{r}_t = \rho_r \hat{r}_{t-1} + (1+\phi_\pi) E_t\{\hat{\pi}_{t+i}\} + \phi_w E_t\{\hat{w}_{t+i}\} + \hat{\varepsilon}_t^r$
7	$\hat{r}_t = \rho_r \hat{r}_{t-1} + (1+\phi_\pi) E_t\{\hat{\pi}_{t+i}\} + \phi_y E_t\{\hat{y}_{t+i}\} + \hat{\varepsilon}_t^r$
8	$\hat{r}_t = \rho_r \hat{r}_{t-1} + (1+\phi_\pi) E_t\{\hat{\pi}_{t+i}\} + \phi_y E_t\{\hat{y}_{t+i}\} + \phi_w E_t\{\hat{w}_{t+i}\} + \hat{\varepsilon}_t^r$

Note: $i \in \{-1; 0; 1\}$ denote the dynamic specification of a policy rule: backward-looking ($i = -1$), contemporaneous ($i = 0$) or forward-looking ($i = 1$).

Source: own elaboration.

2.2. Bayesian estimation

The DSGE model, eqs (2)–(9), with policy rule specification l is a system of linear rational expectations equations of the following form:

[7] Although the Narodowy Bank Polski acts according to direct inflation targeting, the law allows it to target other variables if this approach is consistent with the stabilisation of prices.

[8] We consider all the policy rules that admit one dynamic specification: backward-looking, contemporaneous or forward-looking. Moreover, we assume that each policy rule that responds to inflation may additionally react to real wages or the output gap and may or may not include lagged interest rates.

$$\mathbf{A}(\boldsymbol{\theta}, \boldsymbol{\phi}_l, \boldsymbol{\omega}) E_t\{\mathbf{x}_{t+1}\} + \mathbf{B}(\boldsymbol{\theta}, \boldsymbol{\phi}_l, \boldsymbol{\omega})\mathbf{x}_t + \mathbf{C}(\boldsymbol{\theta}, \boldsymbol{\phi}_l, \boldsymbol{\omega})\mathbf{x}_{t-1} + \mathbf{D}(\boldsymbol{\theta}, \boldsymbol{\phi}_l, \boldsymbol{\omega})\boldsymbol{\varepsilon}_t = 0 \tag{11}$$

where $\mathbf{x}_t = \left[\hat{y}_t, \hat{r}_t, \hat{\pi}_t, \widehat{rmc}_t, \widehat{mrs}_t, \hat{w}_t, \hat{\varepsilon}_t^a, \hat{\varepsilon}_t^b, \hat{\varepsilon}_t^l, \hat{\varepsilon}_t^r\right]'$ is a vector of endogenous variables, $\boldsymbol{\varepsilon}_t = [\eta_t^a, \eta_t^b, \eta_t^l, \eta_t^r]'$ is a vector of innovations in technology and preference shocks, respectively, and $\mathbf{A}(\boldsymbol{\theta}, \boldsymbol{\phi}_l, \boldsymbol{\omega})$, $\mathbf{B}(\boldsymbol{\theta}, \boldsymbol{\phi}_l, \boldsymbol{\omega})$, $\mathbf{C}(\boldsymbol{\theta}, \boldsymbol{\phi}_l, \boldsymbol{\omega})$, $\mathbf{D}(\boldsymbol{\theta}, \boldsymbol{\phi}_l, \boldsymbol{\omega})$ are matrices of which the elements are functions of $\boldsymbol{\theta}, \boldsymbol{\phi}_l, \boldsymbol{\omega}$. These vectors are defined as follows: $\boldsymbol{\theta} = [\delta_c, \delta_l, \theta_p, \theta_w, \rho_a, \rho_b, \rho_l, \sigma_a, \sigma_b, \sigma_l, \sigma_r]' \in \boldsymbol{\Theta}$ is a vector containing structural non--policy estimated parameters, $\boldsymbol{\omega} = [\tau_w, \beta]'$ is a vector of structural non-policy calibrated parameters[9] and $\boldsymbol{\phi}_l = [\rho, \phi_\pi, \phi_y, \phi_w]'$ represents a vector of parameters for monetary policy rule l (see Table 1).

System (11) is solved using generalised Schur decomposition (see Anderson 2008). The solution has the following form:

$$\mathbf{x}_t = \mathbf{F}(\boldsymbol{\theta}, \boldsymbol{\phi}_l, \boldsymbol{\omega})\mathbf{x}_{t-1} + \mathbf{G}(\boldsymbol{\theta}, \boldsymbol{\phi}_l, \boldsymbol{\omega})\boldsymbol{\varepsilon}_t \tag{12}$$

and determines the transition equation in the state space representation of the DSGE model. The state-space model is closed by the measurement equation given by:

$$\mathbf{y}_t = \mathbf{H}\mathbf{x}_t \tag{13}$$

where $\mathbf{y}_t = [\hat{y}_t^{obs}, \hat{r}_t^{obs}, \hat{\pi}_t^{obs}, \hat{w}_t^{obs}]'$ is a vector of observables and $\mathbf{H}$ is a matrix that links observables with their model's counterpart.

The new Keynesian models (12)–(13) are estimated using Bayesian techniques according to the Bayes theorem:[10]

$$p(\boldsymbol{\theta}, \boldsymbol{\phi}_l | \mathbf{Y}_t, \boldsymbol{\omega}) = \frac{p(\boldsymbol{\theta}, \boldsymbol{\phi}_l) L(\mathbf{Y}_t | \boldsymbol{\theta}, \boldsymbol{\phi}_l, \boldsymbol{\omega})}{p(\mathbf{Y}_t)} \tag{14}$$

where $L(\mathbf{Y}_t | \boldsymbol{\theta}, \boldsymbol{\phi}_l, \boldsymbol{\omega})$ is a likelihood function, $p(\boldsymbol{\theta}, \boldsymbol{\phi}_l)$ is a prior distribution, $p(\boldsymbol{\theta}, \boldsymbol{\phi}_l | \mathbf{Y}_t, \boldsymbol{\omega})$ is a posterior distribution and $p(\mathbf{Y}_t)$ represents the marginal data density.

The Bayesian approach emphasises the role of uncertainty about the model and parameters, treating the latter as random variables. According to the likelihood principle, posteriors contain all the relevant information about parameters obtained from data $\mathbf{Y}_t$, including the accuracy of estimates. We refer to the likelihood principle by treating posteriors as a measure of parameter and model uncertainty. The likelihood function may be evaluated by applying the Kalman filter (see DeJong, Dave 2007, among others). The posterior mode and the inverse of the Hessian matrix are found by using numerical procedures. In the final step, the Markov chain Monte Carlo (MCMC) algorithm is applied to draw from the posterior distribution. The algorithm starts with the posterior mode $(\boldsymbol{\theta}^*, \boldsymbol{\phi}_l^*)$ and returns a series $\{\boldsymbol{\theta}_i, \boldsymbol{\phi}_{l,i}\}_{i=0}^N$. The first part of the draws is omitted to ensure that the MCMC algorithm

[9] Although vector $\boldsymbol{\omega}$ may be included in vector $\boldsymbol{\theta}$, our notation distinguishes them and emphasises the common empirical strategy of calibrating some non-identified or poorly identified parameters in applied works (see e.g. Smets, Wouters 2003, 2007).

[10] A more detailed description of the Bayesian estimation of DSGE models can be found in the studies by An and Schorfheide (2007), Fernandez-Villaverde (2010) and Guerron-Quintana and Nason (2012), among others.

draws converge to the true posterior distribution. In the empirical application of our algorithm, we obtain 400,000 draws for two chains in each model and omit the first 300,000 from every chain. After the estimation of all 24 models for l = 1,2,...,24, we perform Bayesian model comparison and choose the model that is preferred by the data.

3. A stochastic dominance approach to policy design

This paper applies the stochastic dominance approach to optimal policy evaluation in the rational expectations models proposed by Górajski and Kuchta (2021). Their approach allows the minimized welfare loss (MWL) distributions and underlying distributions of the optimal policy feedback coefficient (OPFC) to be found by considering the whole posterior distributions of structural parameters and solving the optimal simple rule problem under precommitment in the spirit of Dennis (2004). For the convenience of the reader, below we briefly present a more detailed description of the main idea of their approach. It assumes that the policymaker's objectives at time t are represented by an ad hoc quadratic welfare loss function:

$$L_t\left(\boldsymbol{\theta},\boldsymbol{\phi}_l,l\right)=\left(1-\beta\right)E_t\left\{\sum_{s=0}^{\infty}\beta^s\mathbf{u}'_{\mathbf{t+s}}\mathbf{W}\mathbf{u}_{\mathbf{t+s}}\right\} \tag{15}$$

where $\mathbf{u_t}$ is the vector of central bank target variables, $\mathbf{W}=\operatorname{diag}\left(\lambda_1,\ldots,\lambda_n\right)$ is a diagonal and non--negative matrix of weights and l = 1,2,...,24 indicates the policy rule used to close the model (see Table 1). As shown by Svensson (2010), such a simple mandate is more robust to model and parameter uncertainty (see Debortoli et al. 2019). Svensson (1999) and Dennis (2004) show that, in limiting case $\beta \to 1$, the welfare loss function (31) is given by:

$$L_t\left(\boldsymbol{\theta},\boldsymbol{\phi}_l,l\right)=\sum_{i=1}^{n}\lambda_i\operatorname{var}_{\boldsymbol{\theta},\boldsymbol{\phi}_l,l}\left(u_t^i\right) \tag{16}$$

where u_t^i is the ith component of vector $\mathbf{u_t}$ and $\operatorname{var}_{\boldsymbol{\theta},\boldsymbol{\phi}_l,l}\left(u_t^i\right)$ is the unconditional variance of u_t^i calculated within model (11) with the l policy rule for fixed structural non-policy and policy parameters $\boldsymbol{\theta},\boldsymbol{\phi}_l$. Our choice of target variables is based on the formal derivation of the quadratic welfare loss function proposed by Erceg, Henderson and Levin (2000) and includes inflation rate $\hat{\pi}_t$, output $\hat{y}_t$ and real wage $\hat{w}_t$. To make our optimal policy rules more realistic, we add the interest rate smoothing term $\Delta\hat{r}_t=\hat{r}_t-\hat{r}_{t-1}$ to the welfare loss function. This variable introduces some penalties for significant and quick adjustment in interest rates. As a result, we assume that

$$\mathbf{u_t}=\left[\hat{\pi}_t,\hat{y}_t,\hat{w}_t,\Delta\hat{r}_t\right]' \tag{17}$$

For all the target variables, we specify the weights in the objective function. In our benchmark specification, we set $\lambda_\pi = 1$, $\lambda_y=\lambda_w=\lambda_{\Delta r}=0.5$.

Decision space D consists of the admissible policies (l, $\boldsymbol{\phi}_l$), where l denotes a policy rule from Table 1 and $\boldsymbol{\phi}_l\colon \boldsymbol{\Theta}\to\boldsymbol{\Phi}_l$ is a policy feedback coefficient (PFC) function that maps the parameter space $\boldsymbol{\Theta}$ to the set of policy feedback coefficients $\boldsymbol{\phi}_l$ (see Górajski, Kuchta 2021 for more details). As a result, decision space D consists of all the PFC functions ϕ_l so that DSGE model (11) with policy rule l has a unique solution.

Given DSGE model (11), a set of simple policy rules D, vectors of observables $\mathbf{Y_t}$ and the posterior distribution of structural parameters $p(\boldsymbol{\theta} \mid \mathbf{Y_t}, \boldsymbol{\omega})$, a Bayesian policymaker with SDk preferences solves the problem of the optimal simple macroeconomic policy under parameter uncertainty by finding an SDk-optimal policy rule l^* with feedback coefficients $\boldsymbol{\phi}^*$ such that the corresponding distribution of welfare losses $L_t(\cdot, \boldsymbol{\phi}^*, l^*)$ satisfies

$$L_t\left(\cdot, \boldsymbol{\phi}^*, l^*\right) \leq_{SDk} L_t\left(\cdot, \boldsymbol{\phi}_l, l\right) \text{ for all } \left(l, \boldsymbol{\phi}_l\right) \in D \tag{18}$$

where $\leq_{SDk}$ denotes the inequality between random welfare losses defined by SDk. Thus, the policymaker's optimal decision $(l^*, \boldsymbol{\phi}^*) \in D$ generates the smallest welfare loss distribution with respect to SDk.

The Bayesian policymaker uses the posterior probability distributions of structural model parameters $p(\boldsymbol{\theta} \mid \mathbf{Y_t}, \boldsymbol{\omega})$ as a measure of uncertainty about the structure of the economic model. Therefore, the Bayesian policymaker seeks the best policy rule among those defined in Table 1 by comparing the welfare loss distributions $L_t(\boldsymbol{\theta}, \boldsymbol{\phi}_l, l)$ for all the decisions $(l, \boldsymbol{\phi}_l) \in D$. The probability density functions (PDFs) of $\boldsymbol{\phi}_l(\cdot)$ and $L_t(\cdot, \boldsymbol{\phi}_l, l)$ are inherited from the posterior $p(\boldsymbol{\theta} | \mathbf{Y_t}, \boldsymbol{\omega})$. Theorem 1 of Górajski and Kuchta (2021) gives sufficient conditions for the solution to eq. (18) and shows how the SDk-optimal decision can be identified. As a result, to find the SDk-optimal solution to eq. (18), the Bayesian policymaker seeks, for each l, the optimal policy feedback coefficient (OPFC) function $\boldsymbol{\phi}_l^{min}(\boldsymbol{\theta})$ and the minimised welfare loss (MWL) $L_t^{min}(\boldsymbol{\theta}, l)$ defined by

$$L_t^{min}(\boldsymbol{\theta}, l) = L_t\left(\boldsymbol{\theta}, \boldsymbol{\phi}_l^{\boldsymbol{min}}, l\right) = \min_{\phi_l \in \boldsymbol{\Phi}_l} L_t\left(\boldsymbol{\theta}, \boldsymbol{\phi}_l, l\right) \text{ subject to (11)} \tag{19}$$

for all vectors of structural parameters $\boldsymbol{\theta} \in \boldsymbol{\Theta}$.[11] Then, the SDk-optimal policy rule can be found as $\boldsymbol{\phi}^* = \boldsymbol{\phi}_{l^*}^{\boldsymbol{min}}(\boldsymbol{\theta})$ where l^* is the policy rule that generates the diminutive MWL distribution $L_t^{min}(\cdot, l^*)$ in terms of SDk, that is, $L_t^{min}(\cdot, l^*) \leq_{SDk} L_t^{min}(\cdot, l)$ for all $l = 1,2,..,24$.

Computationally, we find the distribution of OPFCs and MWLs by applying the following steps:[12]

I. Estimate the joint posterior distribution of structural policy and non-policy parameters $p\left(\boldsymbol{\theta}, \boldsymbol{\phi}_{l_e} | \mathbf{Y_t}, \boldsymbol{\omega}\right)$ then integrate $p\left(\boldsymbol{\theta}, \boldsymbol{\phi}_{l_e} | \mathbf{Y_t}, \boldsymbol{\omega}\right)$ over $\boldsymbol{\phi}_{l_e}$ to obtain the posterior PDF of $\boldsymbol{\theta}$, $p(\boldsymbol{\theta} | \mathbf{Y_t}, \boldsymbol{\omega})$, where $l_e \in \{1,2,...,24\}$ indicates DSGE model (11) with policy rule l_e, which has empirical advantages in terms of the marginal data density over all the models. Draw a sequence of vectors $\{\boldsymbol{\theta}_i\}_{i=1}^N$ from the posterior distribution $p(\boldsymbol{\theta} | \mathbf{Y_t}, \boldsymbol{\omega})$.

II. Solve problem (19) for each vector $\boldsymbol{\theta}_i$, $i = 1,2,...,N$ and $l = 1,2,...,24$ and then use the obtained sequences of solutions $\boldsymbol{\phi}_{l,\mathbf{i}}^{\boldsymbol{min}} = \boldsymbol{\phi}_l^{\boldsymbol{min}}(\boldsymbol{\theta}_i)$ and $L_{t,i}^{min}(l) = L_t^{min}(\boldsymbol{\theta}_i, l)$, $i = 1,2,...,N$ to approximate the distributions of $\boldsymbol{\phi}_l^{\boldsymbol{min}}(\cdot)$ and $L_t^{min}(\cdot, l)$ respectively.

[11] We solve the parameterized optimization problem (19) by applying the csminwel procedure that is quasi-Newton method with BFGS update of the estimated inverse hessian (see Sims 1999).

[12] In this paper, we assume that the Bayesian policymaker uses one source of parameter uncertainty. However, a policymaker's decision can modify not only the model structure but also the posterior distribution of the structural parameters and the distribution of welfare losses (see Górajski, Kuchta 2021).

3.1. Comparing the distributions of optimal policy reactions and minimised central loss functions

We use the stochastic dominance relationship between probability distributions to compare the posterior distributions of OPFC and MWL. When, for some $k = 1, 2,...$, the inequality $L_1 \leq_{SDk} L_2$ holds for welfare loss distributions associated with two alternative monetary policy rules, then the welfare losses summarised by L_2 are at least as large as those in the model with a loss function L_1. In our simulations for a given set of optimal and implementable monetary policy rules, we focus on determining the best optimal monetary policy rule, which generates the smallest distribution of welfare losses with respect to SDk ordering. We briefly recall SDk definitions in terms of survival distribution functions. Let $S_X^0(x) = Pr(X > x)$ be the survival distribution function (SDF) of random variable X with the support $(0, L^*)$. For every $k = 1, 2,...$, we define the kth integrals of the SDFs using $S_X^k(x) = \int_x^{L^*} S_X^{k-1}(t)\,dt, x \in [0, L^*]$. The inequality between the kth integrals of the SDFs defines the k-degree stochastic domination; that is, $X_1 \leq_{SDk} X_2$ is equivalent to $S_{X_1}^k(x) \leq S_{X_2}^k(x)$ for all $x \in [0, L^*]$ with strict inequality for some x.

There is comprehensive literature on statistical tests for SDk (see Linton, Maasoumi and Whang 2005; Whang 2019 and references therein). We verify SD1, SD2 or SD3 ordering for each pair of MWL distributions L_1, L_2 by means of a version of the multiple testing procedure proposed by Bishop, Formby and Thistle (1989) (BFT scheme, see also Bennett 2013). Sequentially, for each $k = 1, 2,...$, we consider one of the following null hypotheses: $H_{0,SDk} : L_1 \leq_{SDk} L_2$ or $L_1 =_D L_2$ and $H'_{0,SDk} : L_2 \leq_{SDk} L_1$ or $L_1 =_D L_2$. For each of them, we apply the Kolmogorov-Smirnov (KS)-type test based on the resampling method proposed by Linton, Maasoumi and Whang (2005) with the recentring function (LMW test). To test $H_{0,SDk}$, we calculate the KS distance $\max_{x \in [0, L^*]} \sqrt{\frac{M}{2}} \left(\hat{S}_{L_1}^k(x) - \hat{S}_{L_2}^k(x) \right)$, where $\hat{S}_{L_1}^k$, $\hat{S}_{L_2}^k$ are the empirical survival distribution functions calculated based on the samples of size M from the distributions of L_1 and L_2, respectively. It is worth noting that the high value of the KS distance favours the rejection of the null hypothesis $H_{0,SDk}$. We proceed according to the following version of the BFT scheme:

1. If both $H_{0,SDk}$ and $H'_{0,SDk}$ are accepted, then $L_1 =_D L_2$;
2. If $H_{0,SDk}$ is rejected and $H'_{0,SDk}$ is accepted, then $L_2 \leq_{SDk} L_1$;
3. If $H_{0,SDk}$ is accepted and $H'_{0,SDk}$ is rejected, then $L_1 \leq_{SDk} L_2$;
4. If both $H_{0,SDk}$ and $H'_{0,SDk}$ are rejected, then move on to step $k+1$.

The asymptotic size and power of several versions of the KS-type test are compared by Donald and Hsu (2016).

4. Hallmarks of the optimal monetary policy rules in Poland

In this part of the paper, we apply the approach proposed by Górajski and Kuchta (2021) to find and rank alternative SDk-optimal monetary policy rules in the rational expectations model of the Polish economy. First, a description of the prior distributions and the data used in the estimation are presented. Next, we analyse the posteriors. Finally, we perform a welfare loss analysis to indicate the hallmarks of the optimal monetary policy rules in Poland. We focus on (i) finding the optimal monetary policy rule that minimises the distribution of welfare losses; (ii) measuring the influence of adding the real output and real wages to policy rules on welfare loss distribution; (iii) determining the importance of interest rate smoothing in optimal monetary policy rules; and (iv) comparing the estimated and optimised monetary policy rules. Our simulation result is based on 5,000 random draws from the posterior distribution.

4.1. Priors and data

We estimate the theoretical model for the Polish economy.[13] This part briefly discusses the prior distributions as well as the data used in the estimation. As presented in the previous sections, we divide the parameters into three vectors: vectors of policy parameters $(\boldsymbol{\phi}_l)$ and non-policy estimated parameters $(\boldsymbol{\theta})$ and a vector of calibrated parameters $(\boldsymbol{\omega})$. The first part of Table 2 presents our choice of marginal prior distributions for vectors $\boldsymbol{\phi}_l$ and $\boldsymbol{\theta}$. Our selection seems to be relatively standard considering previous works. We choose beta distributions for all the non-policy parameters that belong to the [0, 1] interval, whereas we select gamma distributions for the positive parameters. Here, we follow Schmitt-Grohe and Uribe (2012). We use a similar scheme for the policy parameters. We impose the beta prior for interest rate smoothing and the gamma prior for policy reaction parameters.

The rest of the structural parameters collected in vector $\boldsymbol{\omega}$ are calibrated. For the discount factor (β), we set the value of 0.99. This implies that the annualised real interest rate in the steady-state equals 4%. The monopolistic wage markup (τ_w) is set at 10%, suggesting that the labour demand elasticity of labour agencies equals -11.

To obtain the posterior distributions, we use quarterly data for the Polish economy from 1995:1 to 2021:3. All the series come from Statistics Poland and Eurostat. We use real GDP per capita as a measure of output, the real wage in the enterprise sector as a measure of real wages,[14] the Consumer Price Index (CPI) quarter to quarter as a measure of inflation and WIBOR 3M (the interbank offer rate) as a measure of the nominal interest rate. Before estimation, the real variables (output and real wage) were expressed as logs, seasonally adjusted using the Tramo-Seats procedure and detrended using a linear filter. The nominal variables were expressed in percentages and seasonally adjusted (except the interest rate). Next, they were divided into two periods: from 1995:1 to 2003:4, from which we exclude the quadratic trend, and from 2004:1 to 2021:3, in which we demean both variables. These transformations are justified by the substantial disinflation period in Poland after the transition from a centrally planned to a market economy and the behaviour of the central bank's inflation target in Poland. Inflation decreased substantially from the beginning of the sample until the end of 2003, and after this period it became constant.[15]

[13] It is worth noting that our model is estimated in previous studies by Rabanal and Rubio-Ramirez for the U.S. (2005) and for the euro area (2008) as well as Kuchta (2014) and Baranowski and Kuchta (2015) for the Polish economy.

[14] We deflate the nominal series using the CPI index with the base 2015:1.

[15] It is worth noting that the theoretical model assumes that the inflation target is constant over time and consistent with a zero inflation steady state.

Table 2

Prior and posterior distributions

Vector	Parameter		Prior				Posterior		
	Name	Symbol	Range	Type	Mean	Standard deviation	5%	Mean	95%
θ	Relative risk aversion	δ_c	(0; ∞)	Gamma	1.25	0.5	1.30	1.89	2.41
	Inverse of Frisch labour elasticity	δ_l	(0; ∞)	Gamma	1.25	0.5	0.10	0.26	0.40
	Price stickiness	θ_p	(0; 1)	Beta	0.5	0.2	0.85	0.87	0.90
	Wage stickiness	θ_w	(0; 1)	Beta	0.5	0.2	0.59	0.65	0.71
	Autoregressive parameter – technological shock	ρ_a	(0; 1)	Beta	0.5	0.2	0.19	0.27	0.34
	Autoregressive parameter – preference shock	ρ_b	(0; 1)	Beta	0.5	0.2	0.72	0.79	0.86
	Autoregressive parameter – labour supply shock	ρ_l	(0; 1)	Beta	0.5	0.2	0.05	0.16	0.28
	Standard deviation – technological shock	σ_a	(0; ∞)	Gamma	0.1	0.05	0.12	0.20	0.28
	Standard deviation – labour supply shock	σ_l	(0; ∞)	Gamma	0.1	0.05	0.15	0.24	0.33
	Standard deviation – preference shock	σ_b	(0; ∞)	Gamma	0.1	0.05	0.04	0.05	0.06
	Standard deviation – monetary policy shock	σ_r	(0; ∞)	Gamma	0.1	0.05	0.002	0.0028	0.003
$\boldsymbol{\phi}_l$	Monetary policy rule – interest rate smoothing	ρ	(0; 1)	Beta	0.5	0.2	0.90	0.95	0.99
	Monetary policy rule – reaction to inflation	ϕ_π	(0; ∞)	Gamma	0.5	0.25	0.03	0.11	0.19
	Monetary policy rule – reaction to output	ϕ_y	(0; ∞)	Gamma	0.125	0.05	-	-	-
	Monetary policy rule – reaction to real wage	ϕ_w	(0; ∞)	Gamma	0.1	0.05	-	-	-

Source: own computations.

4.2. Posteriors

We estimate all 24 models, which differ in monetary policy rules (see Table 1). Next, we perform a Bayesian model comparison and find that the data prefer the model with forward-looking rule no. 5.[16] The posterior distributions for this model are presented in the second part of Table 2. We report the posterior mean and the 90% highest posterior density (HPD) interval. The posterior estimates indicate a high level of price stickiness and a moderate level of wage stickiness. The domination of price stickiness rather than wage stickiness seems to be counterintuitive. However, it is a permanent feature of the DSGE model with constant returns to scale in production and a Calvo scheme of price stickiness (see Smets, Wouters 2003). Focusing on posterior means, the average duration of price contracts lasts 7.7 quarters. Although our estimates are higher than micro-evidence,[17] they are comparable with those previously found in the literature. For example, Leszczyńska-Paczesna (2020) estimate a two--sector DSGE model and find significant differences in the price stickiness between these sectors. For the food and energy production sectors, the average duration of price contracts is 3.2 quarters, whereas, for the core inflation sector, it is 20 quarters.[18] Moreover, our estimates are fairly close to the previous estimates obtained from Erceg, Henderson and Levin's (2000) model for the Polish economy (see Kuchta 2014). The average duration of wage contracts lasts 2.9 quarters. Although these estimates may suggest weak rigidity, the importance of wage stickiness as part of the DSGE model is strongly supported by the empirical results.[19] Moreover, similar low wage stickiness is found in previous studies on the Polish economy. For example, Kolasa (2009) estimate the two-country DSGE model for Polish and euro area data. The estimates of the Calvo wage stickiness parameter imply that the average duration of wage contracts equals 2.6 quarters. Finally, the estimates of monetary policy rule parameters indicate a limited reaction to inflation and a substantial effect of interest rate smoothing. It is worth noting that monetary policy rule no. 5 does not contain a reaction to the output gap and inflation. Thus, we do not find empirical evidence that the actual nominal interest rate is driven by these variables.

4.3. Welfare loss analysis

This section presents the results of the welfare analysis performed by employing the stochastic dominance approach (see Section 3). Under parameter uncertainty, we verify several hypotheses on the features of the optimal monetary policy in Poland by comparing 24 simple monetary policy rules (see Table 1).[20] Figure 1 shows the highest density intervals (HDIs) for all 24 analysed policy rules

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16 The Bayesian model comparison is performed in Appendix B.

17 Macias and Makarski (2013) investigate the average duration of price contracts using microdata for the Polish economy. They find that the average duration of price contracts equals 11 months and is higher than in the U.S. and lower than in the euro area.

18 It is worth noting that Leszczyńska-Paczesna (2020) assumes that the share of core inflation in the CPI index is 0.6.

19 Rabanal and Rubio-Ramirez (2005) compare different specifications of a small-scale DSGE model using data for the U.S. They find that an Erceg, Henderson and Levin-type model is preferred by the data over the models omitting wage stickiness even if price indexation is introduced. This observation is confirmed in the case of the euro area (Rabanal, Rubio-Ramirez 2008) and Poland (Kuchta 2014). Moreover, Smets and Wouters (2007) investigate the empirical importance of nominal and real rigidities in the medium-scale DSGE model in the spirit of Christiano, Eichenbaum and Evans (2005). Their results also suggest the importance of wage stickiness. Finally, similar results are found by Adolfson et al. (2007) in an open-economy DSGE model.

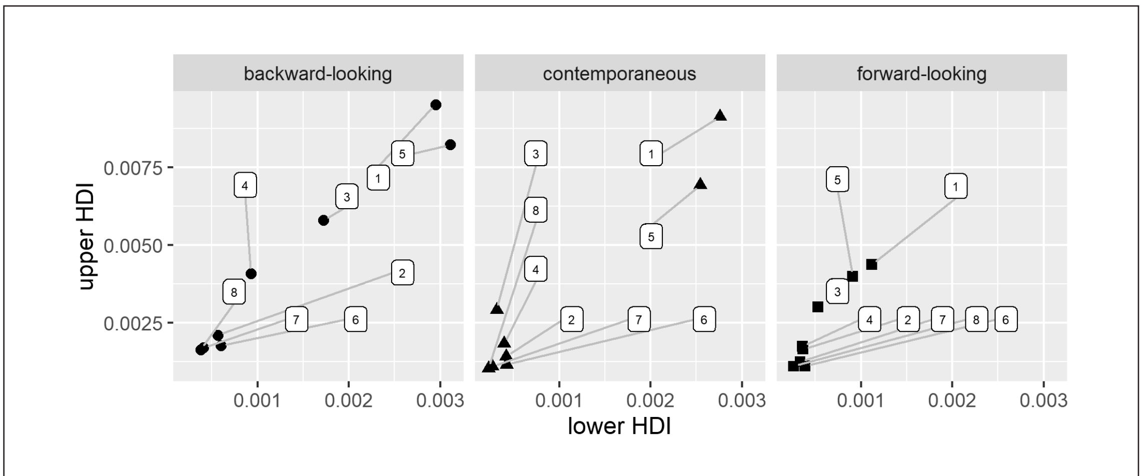
20 All considered rules omit the monetary policy shock.

grouped by their dynamic specification. The horizontal and vertical axes measure the lower and upper HDI intervals, respectively. The closer a point is to the origin, the smaller the welfare losses are, whereas the distance from the identity line measures the uncertainty in welfare losses. We observe that contemporaneous and forward-looking rules generate smaller losses than corresponding backward-looking policies. Contemporaneous and forward-looking rules 7 and 8 are the leaders in making the welfare losses close to zero levels. We compare all the pairs of the welfare loss distributions utilising the BFT scheme with the KS-type test, and we receive a precise result that the SD1-optimal policy rules allow contemporaneous dynamic specification:[21]

$$\hat{r}_t = \underset{[1.06,1.22]}{1.13}\ \hat{r}_{t-1} + \underset{[1.53,1.62]}{1.57}\ \hat{\pi}_t + \underset{[1.08,1.53]}{1.29}\ \hat{y}_t + \underset{[0.71,1.05]}{0.88}\ \hat{w}_t \qquad (20)$$

The best SD1-optimal contemporaneous monetary policy rule (20) offers the following features. First, it is a super-inertial interest rate rule (see Giannoni, Woodford 2002; Schmitt-Grohe, Uribe 2004). Thus, the optimal monetary policy instrument is a function of lagged values of the policy instrument, and the persistence parameter ρ_r takes values higher than 1. Second, we receive a moderate reaction to inflation, which is consistent with the results of Taylor (1993). Third, we report that the best SD1-optimal monetary policy rule shows a moderate reaction to both the output gap and the real wages.

Figure 1
HDIs of MWL distributions for optimal Taylor - type rules



Notes: 90% HDIs for all 24 Taylor rules (see Table 1).
Source: own computations.

[21] In addition, we estimate a new version of the model on a sample from 1995:1 to 2019:4 that does not cover the period of the COVID-19 pandemic and re-rank the rules based on the proposed stochastic dominance criterion. The best specification does not change and the contemporaneous policy rule no. 8 is the SD1-optimal policy rule, taking the following form:

$$\hat{r}_t = \underset{[1.03,1.14]}{1.09}\ \hat{r}_{t-1} + \underset{[1.56,1.62]}{1.59}\ \hat{\pi}_t + \underset{[1.00,1.43]}{1.22}\ \hat{y}_t + \underset{[0.69,0.99]}{0.85}\ \hat{w}_t$$

However, the distributions of the OPFC shift. The SD1-optimal rule after the outbreak of the COVID-19 pandemic is less responsive to inflation and at the same time responds more strongly to changes in the output gap and real wages.

4.4. The influence of real variables on welfare loss and SDk-optimal monetary policy rules

This part evaluates the consequences of using different measures of economic fluctuations in optimal monetary policy rules. The first part of our analysis compares the strict inflation-responsive instrument policy rules[22] (Model 1 in Table 4), in which the interest rate responds only to inflation, with the flexible inflation-responsive instrument policy rules (Model 2 in Table 4). Using the latter, the central bank chooses the nominal interest rate as a function of the inflation rate and measures of economic activity, for example the output and real wage. The real wage may be seen as an alternative and more connected with the labour market measures of business cycle fluctuations. The results of the comparison are presented in Table 4. Observe that the distinction between welfare loss distributions is performed by SD1 ordering. Therefore, the decision to include real variables in the policy rule is independent of the policymaker's attitude towards risk. Intuitively, and consistent with an assessment provided by Rudebusch and Svensson (1999), the strict inflation-responsive instrument rule cannot reduce welfare loss to the levels comparable with an alternative monetary policy rule. The results of the LMW tests indicate that the welfare losses implied by rules 1 and 5 are statistically higher than the losses obtained from rules 2–4 and 6–8, respectively. These results hold for backward-looking, forward-looking and contemporaneous rules. Consequently, the monetary policy rule that reacts only to inflation is suboptimal.

In the second dimension, we analyse the effect of adding different measures of business cycle activity to the augmented Taylor-type rules. The results are presented in Table 5. The application of the BFT scheme with LMW tests for SD2 allows us to rank the augmented Taylor-type rules with reference to the risk-averse attitude of the policymaker. For backward-looking rules, incorporating real wages into the optimal monetary policy rule (rule no. 2) causes statistically lower welfare loss than rules including the output (rule no. 3), even if the real wages are also a reaction variable (rule no. 4). These results do not hold for rules with an interest rate smoothing incentive. Incorporating this mechanism into the optimal monetary policy rules causes the rule with output (rule no. 7) to generate statistically lower welfare loss than the rule reacting to the real wage (rule no. 6). However, including both real variables in the policy rule limits welfare loss even more.

A similar pattern emerges when contemporaneous rules are considered. Incorporating the real wage into a policy rule without interest rate smoothing (rule no. 2) causes a statistically lower level of welfare loss than using a rule that includes only output (rule no. 3). Additionally, rule no. 2 implies a statistically lower level of welfare loss than rule no. 4. Notably, the distinction between these two pairs of rules requires the imposition of a risk-averse attitude on policymaker preferences (SD2 preferences).[23] Moreover, considering both variables (rule no. 4) causes a statistically lower level of welfare loss than reacting only to output (rule no. 3). These results do not hold for rules with interest rate smoothing. The rule with output indicates a statistically lower level of welfare loss than the rule reacting only to the real wage, whereas the broadest rule (rule no. 8) generates welfare loss that is lower than those generated by rule no. 6 and rule no. 7.

[22] Here, we follow Svensson (2002) in recognising the difference between inflation-targeting and instrument rules.

[23] This is a clear advantage of Górajski and Kuchta's (2021) method over the fixed-parameter approach. Besides comparing the whole distributions, it enables us to recognise different policymaker preferences towards welfare losses that are required to distinguish between pairs of rules.

Table 4

Results of the LMW tests for measuring the influence of real variables on the distributions of minimised welfare loss

Type of rule	Model 1					Model 2					Policymaker preferences
	No. of rule	Welfare loss L_1				No. of rule	Welfare loss L_2				
		Mean	Median	5%	95%		Mean	Median	5%	95%	
Backward-looking (i = -1)	1	6.28	5.96	3.01	9.55	2	1.38	1.28	0.60	2.13	SD1
	1	6.28	5.96	3.01	9.55	3	3.70	3.52	1.69	5.75	SD1
	1	6.28	5.96	3.01	9.55	4	2.49	2.31	0.92	4.07	SD1
	5	5.72	5.49	3.14	8.25	6	1.20	1.14	0.62	1.79	SD1
	5	5.72	5.49	3.14	8.25	7	1.05	0.98	0.40	1.69	SD1
	5	5.72	5.49	3.14	8.25	8	1.01	0.94	0.37	1.63	SD1
Contemporaneous (i = 0)	1	5.88	5.55	2.73	9.10	2	0.93	0.86	0.42	1.43	SD1
	1	5.88	5.55	2.73	9.10	3	1.60	1.41	0.35	2.93	SD1
	1	5.88	5.55	2.73	9.10	4	1.04	0.92	0.39	1.82	SD1
	5	4.77	4.56	2.50	6.88	6	0.80	0.76	0.44	1.18	SD1
	5	4.77	4.56	2.50	6.88	7	0.66	0.60	0.26	1.10	SD1
	5	4.77	4.56	2.50	6.88	8	0.62	0.57	0.25	1.06	SD1
Forward-looking (i = 1)	1	2.70	2.52	1.12	4.37	2	1.04	0.95	0.38	1.67	SD1
	1	2.70	2.52	1.12	4.37	3	1.73	1.55	0.50	2.98	SD1
	1	2.70	2.52	1.12	4.37	4	1.05	0.94	0.35	1.73	SD1
	5	2.45	2.21	0.92	3.99	6	0.75	0.71	0.39	1.12	SD1
	5	2.45	2.21	0.92	3.99	7	0.76	0.71	0.34	1.26	SD1
	5	2.45	2.21	0.92	3.99	8	0.66	0.61	0.27	1.11	SD1

Notes:
Each row contains a comparison of MWLs L_1 and L_2 in Model 1 and Model 2, and statistics about the distributions of L_1 and L_2, all figures being multiplied by 10^3. We apply the LMW test to each of them; we then proceed according to the BFT scheme (see Section 3). The shaded cells show a model that generates a statistically smaller MWL in terms of SDk policymaker preference, with decisions on the significance level $\alpha = 0.05$.

Source: own computations.

A different picture emerges for forward-looking rules without interest rate smoothing. In this group of rules, the rule with the real wage (rule no. 2) allows for a statistically lower level of welfare loss than the standard Taylor rule (rule no. 3). However, the rule with both variables (rule no. 4) generates the same welfare loss distribution as rule no. 2 while implying a smaller welfare loss than rule no. 3. Adding the interest rate smoothing mechanism changes the ranking of the policy rules. When imposing a risk--averse attitude on policymaker preferences (SD2 preferences), the rule reacting only to the real wage (rule no. 6) is able to generate lower welfare loss distribution than the rule with an output gap (rule no. 7). However, rule no. 8 dominates all the forward-looking rules.

Table 5
Results of the LMW tests for measuring the influence of real variables on the distributions of minimised welfare loss

Type of rule	Model 1					Model 2					Policymaker preferences
	No. of rule	Welfare loss L_1				No. of rule	Welfare loss L_2				
		Mean	Median	5%	95%		Mean	Median	5%	95%	
Backward-looking (i = -1)	2	1.38	1.28	0.60	2.13	3	3.70	3.52	1.69	5.75	SD1
	2	1.38	1.28	0.60	2.13	4	2.49	2.31	0.92	4.07	SD1
	3	3.70	3.52	1.69	5.75	4	2.49	2.31	0.92	4.07	SD1
	6	1.20	1.14	0.62	1.79	7	1.05	0.98	0.40	1.69	SD1
	6	1.20	1.14	0.62	1.79	8	1.01	0.94	0.37	1.63	SD1
	7	1.05	0.98	0.40	1.69	8	1.01	0.94	0.37	1.63	SD1
Contemporaneous (i = 0)	2	0.93	0.86	0.42	1.43	3	1.60	1.41	0.35	2.93	SD2
	2	0.93	0.86	0.42	1.43	4	1.04	0.92	0.39	1.82	SD2
	3	1.60	1.41	0.35	2.93	4	1.04	0.92	0.39	1.82	SD1
	6	0.80	0.76	0.44	1.18	7	0.66	0.60	0.26	1.10	SD1
	6	0.80	0.76	0.44	1.18	8	0.62	0.57	0.25	1.06	SD1
	7	0.66	0.60	0.26	1.10	8	0.62	0.57	0.25	1.06	SD1
Forward-looking (i = 1)	2	1.04	0.95	0.38	1.67	3	1.73	1.55	0.50	2.98	SD1
	2	1.04	0.95	0.38	1.67	4	1.05	0.94	0.35	1.73	$=_D$
	3	1.73	1.55	0.50	2.98	4	1.05	0.94	0.35	1.73	SD1
	6	0.75	0.71	0.39	1.12	7	0.76	0.71	0.34	1.26	SD2
	6	0.75	0.71	0.39	1.12	8	0.66	0.61	0.27	1.11	SD1
	7	0.76	0.71	0.34	1.26	8	0.66	0.61	0.27	1.11	SD1

Notes:
Each row contains a comparison of MWLs L_1 and L_2 in Model 1 and Model 2, and statistics about the distributions of L_1 and L_2, all figures being multiplied by 10^3. We apply the LMW test and then proceed according to the BFT scheme (see Section 3). The shaded cells show a model that generates a statistically smaller MWL in terms of SDk policymaker preferences, with decisions on the significance level $\alpha = 0.05$.
Source: own computations.

In conclusion, our analysis shows that, under parameter uncertainty, the strict inflation-responsive instrument rules seem to be a rather non-optimal strategy for policymakers compared with flexible inflation-responsive instrument rules. This result appears to be quite intuitive, since the central bank is usually interested in observing and reacting to a broad set of macroeconomic indicators. Moreover, the choice of actual variables that policymakers should include in monetary policy rules is not always obvious, and rules that react only to inflation and the output gap do not necessarily limit the welfare loss more than rules that respond only to inflation and real wages. The advantage of reacting to the output gap rather than the real wage depends on the dynamic specification and the existence of interest rate smoothing.

4.5. The importance of interest rate smoothing in SDk-optimal monetary policy rules

This part analyses the consequences of introducing interest rate smoothing into the optimal monetary policy rule. Interest rate smoothing may be seen as a sign of cautiousness in monetary policy since it allows for a gradual response to the nominal interest rate. Table 6 presents the results. We find an unambiguous result in that interest rate smoothing substantially limits the welfare loss distributions. This is true for backward-looking, forward-looking and contemporaneous rules and for all the reaction variables. It is worth noting that this result is confirmed by the stochastic dominance approach.

Table 6

Results of the LMW tests for measuring the influence of interest rate smoothing on the distributions of minimised welfare loss

Type of rule	Model 1					Model 2					Policymaker preferences
	No. of rule	Welfare loss L_1				No. of rule	Welfare loss L_2				
		Mean	Median	5%	95%		Mean	Median	5%	95%	
Backward-looking (i = -1)	1	6.28	5.96	3.01	9.55	5	5.72	5.49	3.14	8.25	SD1
	2	1.38	1.28	0.60	2.13	6	1.20	1.14	0.62	1.79	SD1
	3	3.70	3.52	1.69	5.75	7	1.05	0.98	0.40	1.69	SD1
	4	2.49	2.31	0.92	4.07	8	1.01	0.94	0.37	1.63	SD1
Contemporaneous (i = 0)	1	5.88	5.55	2.73	9.10	5	4.77	4.56	2.50	6.88	SD1
	2	0.93	0.86	0.42	1.43	6	0.80	0.76	0.44	1.18	SD1
	3	1.60	1.41	0.35	2.93	7	0.66	0.60	0.26	1.10	SD1
	4	1.04	0.92	0.39	1.82	8	0.62	0.57	0.25	1.06	SD1
Forward-looking (i = 1)	1	2.70	2.52	1.12	4.37	5	2.45	2.21	0.92	3.99	SD1
	2	1.04	0.95	0.38	1.67	6	0.75	0.71	0.39	1.12	SD1
	3	1.73	1.55	0.50	2.98	7	0.76	0.71	0.34	1.26	SD1
	4	1.05	0.94	0.35	1.73	8	0.66	0.61	0.27	1.11	SD1

Notes:
Each row contains a comparison of MWLs L_1 and L_2 in Model 1 and Model 2, and statistics about the distributions of L_1 and L_2, all figures being multiplied by 10^3. We apply the LMW test and then proceed according to the BFT scheme (see Section 3). The shaded cells show a model that generates a statistically smaller MWL in terms of SDk policymaker preferences, with decisions on the significance level α = 0.05.

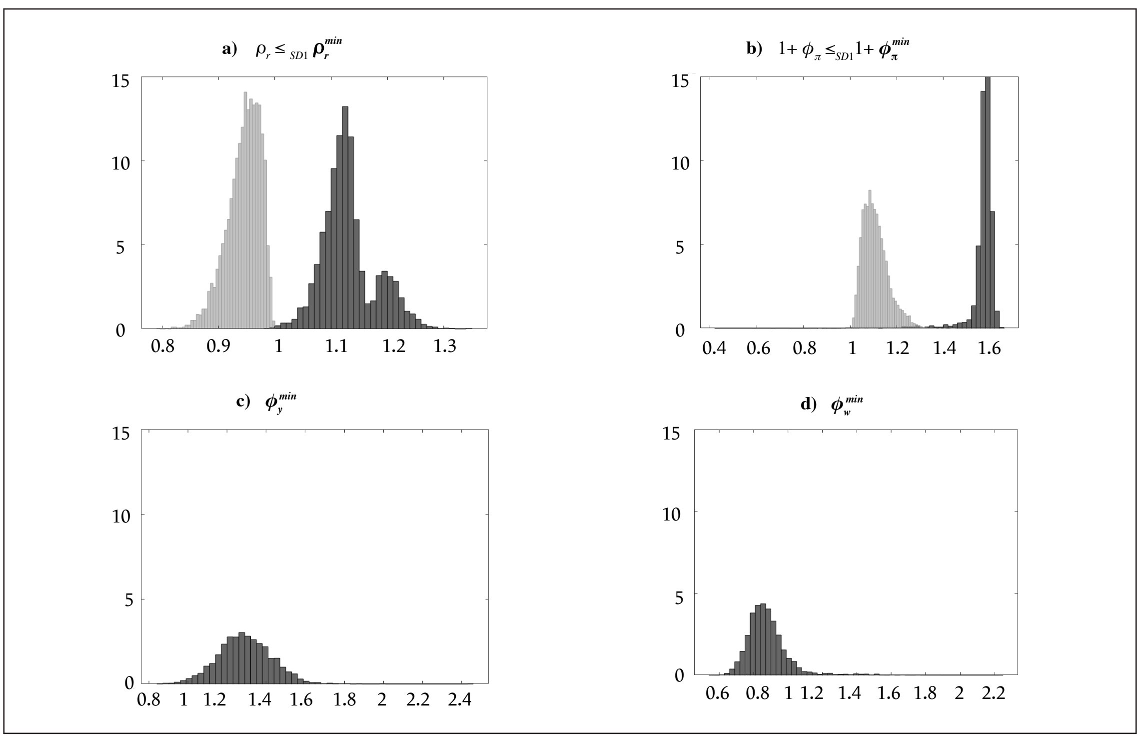
Source: own computations.

In Appendix C, we repeat the computations for Table 6, applying the optimised simple rule approach (see Dennis 2004). Following this approach, we do not consider the whole posterior distribution of structural parameters and assume that vector $\boldsymbol{\theta}$ is fixed on the posterior median. We show that, under this assumption, rule no. 1 generates smaller welfare losses than rule no. 5. Therefore, the stochastic dominance approach may result in a different ranking from the optimised simple rule approach.

4.6. Comparison of estimated and optimised monetary policy rules

This section compares the distributions of parameters describing the central bank's reactions resulting from the SD1-optimal monetary policy rule and the best actual/empirical forward-looking rule no. 5 (see Figure 2). Furthermore, for these two rules, we perform a statistical comparison between the distributions of the minimised and empirical welfare loss and the standard deviation of the target variables (see Figure 3). In particular, we check whether there is a significant difference between the probability distributions of inflation volatilities derived from the empirical model and those derived from the model with the SD1 optimal rule.

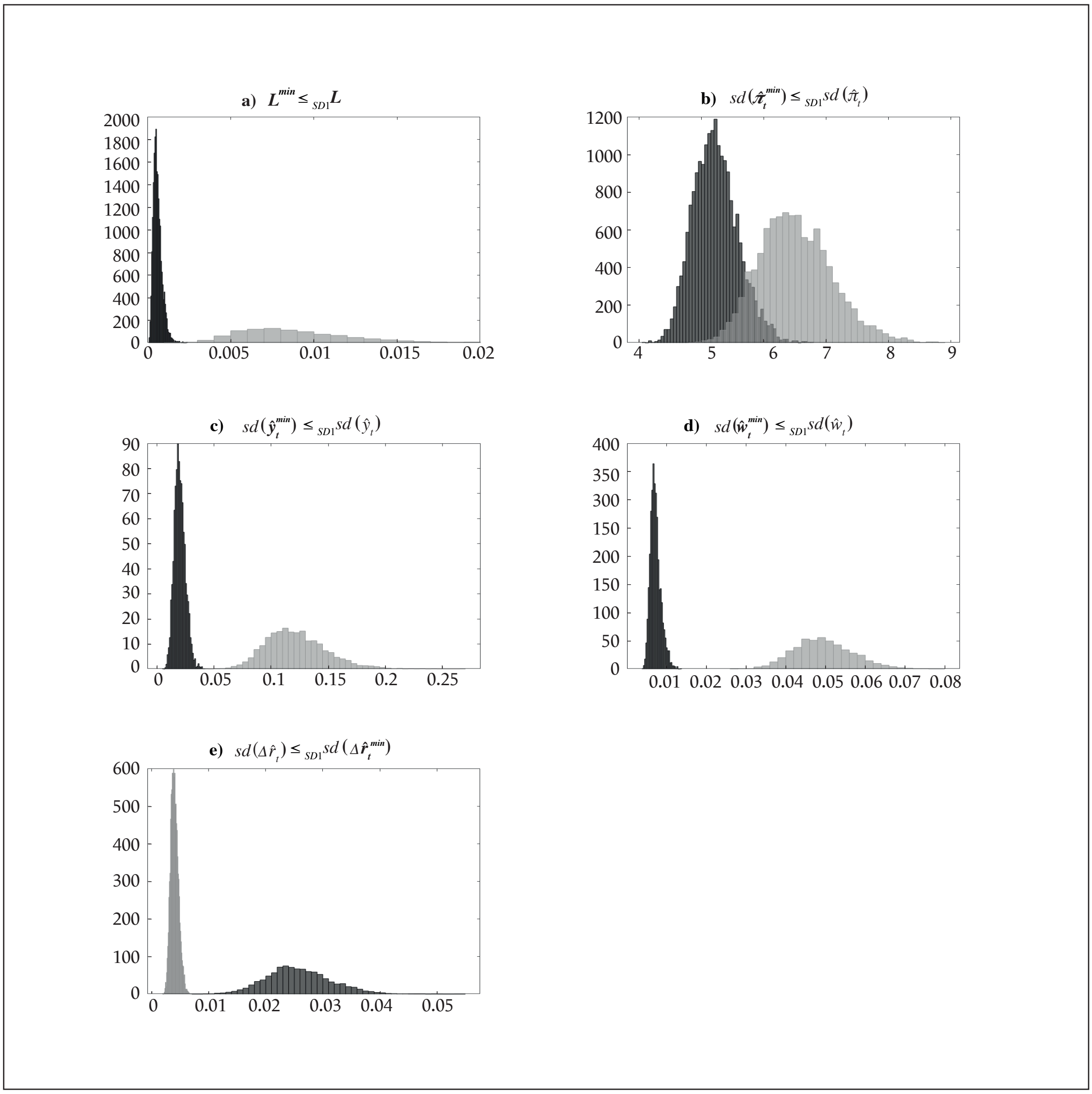
Figure 2
Probability density functions of parameters in the actual forward-looking rule no. 5 and the SD1-optimal contemporaneous rule no. 8



Notes:
The relation between parameters is confirmed using the LMW test, with decisions on the significance level α = 0.05. The actual forward-looking monetary policy rule, $\hat{r}_t = \rho_r \hat{r}_{t-1} + \left(1+\phi_\pi\right) E_t \hat{\pi}_{t+1}$, is in the grey shaded area, and the best SD1--optimal forward-looking monetary policy rule, $\hat{r}_t = \rho_r^{min} \hat{r}_{t-1} + \left(1+\phi_\pi^{min}\right) \hat{\pi}_t + \phi_y^{min} \hat{y}_t + \phi_w^{min} \hat{w}_t$, is in the black shaded area.
Source: own computations.

Figure 3

Probability density functions of welfare losses and standard deviations of target variables



Notes:

The relation between the standard deviations (sd) of the target variables is confirmed using the LMW test, with decisions on the significance level α = 0.05. The actual forward-looking monetary policy rule, $\hat{r}_t = \rho_r \hat{r}_{t-1} + (1+\phi_\pi) E_t \hat{\pi}_{t+1}$, is in the grey shaded area, and the best SD1-optimal forward-looking monetary policy rule, $\hat{r}_t = \rho_r^{min} \hat{r}_{t-1} + (1+\phi_\pi^{min})\hat{\pi}_t + \phi_y^{min}\hat{y}_t + \phi_w^{min}\hat{w}_t$, is in the black shaded area.

Source: own computations.

The LMW test confirms that the SD1-optimal rule generates significantly smaller welfare loss than its empirical counterpart. For this rule, the median of MWL is more than six times lower than the median resulting from the application of the empirical rule (see Figure 3 a). To achieve this welfare loss reduction, the optimal central bank should increase all its reactions to lagged interest rates and inflation and respond positively to the output and real wage[24] (see Figure 2 a–b).

One of the fundamental questions about implementing the optimal monetary policy rules is whether it can sufficiently reduce the volatility of inflation, which is essential for central banks following strict inflation targeting. Figure 3 b–e compares the distributions of volatility of the target variables, measured using the standard deviation. The LMW tests show that, in the model with the SD1--optimal policy, the distributions of volatilities of all the endogenous variables are significantly smaller than those of their empirical counterparts. Hence, the best optimal rule can control fluctuations in inflation, output and wages. However, the price that the SD1-optimal central bank must bear to reduce the fluctuation of these variables is a marked increase in the volatility of change in the policy instrument (see Figure 3e).

5. Conclusions

This study applies the stochastic dominance approach proposed by Górajski and Kuchta (2021) to optimal policy rule evaluation in a DSGE model of the Polish economy. This approach assumes that policymakers measure parameter uncertainty by posterior distributions obtained using the Bayesian inference performed on the data. In contrast to previous works, we derive the whole distributions of optimal policy feedback parameters and implied minimised welfare losses. Minimisation of the welfare loss function is performed under the assumption that the policymaker faces uncertainty regarding the model's structural parameters and pre-commits to a policy rule with optimal feedback coefficients. To compare optimised rules, policymakers use SDk orderings. Contrary to other methods of comparing random variables, SDk has a direct economic interpretation and allows competing rules to be distinguished even if the first central moments of welfare losses are quite close.

We use an estimated version of the sticky price and wages DSGE model to ask several questions about Poland's optimal monetary policy stance. We perform a welfare analysis for the set of 24 feasible policy rules and find that a contemporaneous rule with interest rate smoothing that responds to all the target variables (inflation, output and real wage) is the best SD1-optimal simple rule for all Bayesian policymakers with first-degree stochastic dominance preferences. We compare two strategies for conducting monetary policy: strict and flexible inflation-responsive instrument rules. Firstly, we show that the strict inflation-responsive instrument rules seem to be a rather non-optimal strategy for policymakers who take parameter uncertainty into account. Secondly, we compare several pairs of monetary policy rules and show that the most popular standard Taylor rule does not necessarily generate the smallest welfare loss. Thirdly, we prove that interest rate smoothing substantially limits the welfare loss distributions. Finally, we compare the estimated and optimised interest rate rules and find significant differences between posteriors obtained from data and optimal distributions. We show that the optimal interest rate rule exhibits a stronger reaction to inflation than those found in the data. A more aggressive monetary policy will allow the reduction of variability in inflation, the output gap and real wages but will increase the volatility of the monetary policy instrument.

[24] We recall that the results of the model comparison reveal that the empirical rule for our sample does not react to the output and real wage.

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Appendix A. Theoretical model

This appendix provides a short description of our theoretical model. Our economy consists of final-good firms, intermediate-good firms, labour agencies and households. It is assumed that firms are indexed by $j \in [0;1]$, whereas households are indexed by $i \in [0;1]$.

The final good, Y_t, consists of an infinite number of non-perfectly substitutive intermediate goods, $Y_t(j)$, and is produced according to the following technology (Dixit, Stiglitz 1977):

$$Y_t = \left[\int_0^1 Y_t(j)^{\frac{1}{1+\tau_p}} dj \right]^{1+\tau_p} \tag{A.1}$$

where $\tau_p > 0$ is the monopolistic markup on the goods market. A representative final-good firm maxi--mises its profits and treats the price of final good P_t and the price of intermediate j-good $P_t(j)$ as given.

Thus, the optimal demand function is provided by:

$$Y_t(j) = \left(\frac{P_t(j)}{P_t} \right)^{-\frac{1+\tau_p}{\tau_p}} Y_t \tag{A.2}$$

for all $j \in [0, 1]$ and the implied general level of prices is given by:

$$P_t = \left[\int_0^1 P_t(j)^{-\frac{1}{\tau_p}} dj \right]^{-\tau_p} \tag{A.3}$$

We assume that every intermediate good j is produced by a monopolistically competitive firm using only labour inputs L_t^j according to the following technology:

$$Y_t(j) = \varepsilon_t^a L_t^j \tag{A.4}$$

where ε_t^a represents the level of technology that evolves according to the stationary AR(1) process:

$$\ln \varepsilon_t^a = (1-\rho_a)\ln \varepsilon^a + \rho_a \ln \varepsilon_{t-1}^a + \sigma_a \eta_t^a ; \eta_t^a \sim i.i.d.N(0;1) \tag{A.5}$$

where $\rho_a \in [0;1]$ is an autoregressive parameter and $\sigma_a > 0$ represents the standard deviation of a technological shock.

We assume that each firm hires labour in a perfectly competitive labour market and pays a real wage, w_t. Under the technology of production (A.4), the real marginal cost $RMC_t(j)$ does not depend on the level of output:

$$RMC_t(j) = \frac{w_t}{\varepsilon_t^a} \tag{A.6}$$

Following Calvo (1983) and Yun (1996), it is assumed that, in each period, only a randomly chosen part of intermediate firms, $1-\theta_p \in (0;1)$, can reoptimise their price. Each firm chooses the price to maximise the expected sum of discounted profits:

$$E_t\left\{\sum_{s=0}^{\infty}(\beta\theta_p)^s\frac{\lambda_{t+s}}{\lambda_t}Y_{t+s}(j)\left[\frac{P_t(j)}{P_{t+s}}-RMC_{t+s}(j)\right]\right\} \tag{A.7}$$

subject to demand function (A.2), where $\beta^s\frac{\lambda_{t+s}}{\lambda_t}$ is the stochastic discount factor and E_t is the rational expectations operator. The rest of the prices remain constant. The first-order condition is given by:

$$E_t\left\{\sum_{s=0}^{\infty}(\beta\theta_p)^s\frac{\lambda_{t+s}}{\lambda_t}Y_{t+s}^*(j)\left[(1+\tau_p)RMC_{t+s}(j)-\frac{P_t^*(j)}{P_{t+s}}\right]\right\}=0 \tag{A.8}$$

where:

$$Y_{t+s}^*(j)=\left(\frac{P_t^*(j)}{P_{t+s}}\right)^{-\frac{1+\tau_p}{\tau_p}}Y_{t+s} \tag{A.9}$$

Condition (A.8) shows that the intermediate-good firm chooses the price to equate the expected average future marginal revenues to the future expected markups over the real marginal cost (Schmitt--Grohe, Uribe 2004, p. 13). Since all re-optimising firms face an identical demand curve (A.2) and real marginal cost (A.6), they will choose the same price. This property allows us to express the price of a final good (A.3) as follows:

$$P_t=\left[(1-\theta_p)P_t^{*\,-\frac{1}{\tau_p}}+\theta_p P_{t-1}^{-\frac{1}{\tau_p}}\right]^{-\tau_p} \tag{A.10}$$

Labour services, L_t, are provided by an agency that aggregates the heterogeneous labour services, $L_t(i)$, delivered by households into homogeneous input using the following technology:

$$L_t=\left[\int_0^1 L_t(i)^{\frac{1}{1+\tau_w}}di\right]^{1+\tau_w} \tag{A.11}$$

where $\tau_w > 0$ is the monopolistic markup on the labour market. The optimal demand for labour is represented by:

$$L_t(i)=\left(\frac{w_t(i)}{w_t}\right)^{-\frac{1+\tau_w}{\tau_w}}L_t \tag{A.12}$$

for all $i \in [0; 1]$, where $w_t(i)$ is the real wage of household i and the real wage w_t is given by:

$$w_t = \left[\int_0^1 w_t(i)^{-\frac{1}{\tau_w}} di \right]^{-\tau_w} \tag{A.13}$$

Each household tends to maximise its lifetime utility, described by:

$$E_t \left\{ \sum_{k=0}^{\infty} \beta^k \varepsilon_{t+k}^b \left[\frac{C_{t+k}(i)^{1-\delta_c}}{1-\delta_c} - \varepsilon_t^l \frac{L_{t+k}(i)^{1+\delta_l}}{1+\delta_l} \right] \right\} \tag{A.14}$$

where $C_t(i)$ is consumption, $\beta \in (0; 1)$ is a subjective discount factor, $\delta_c > 0$ represents the relative risk aversion parameter, $\delta_l > 0$ is the inverse of Frisch's labour elasticity, ε_t^b denotes a preference shock and ε_t^l is a labour supply shock. Both of the shocks follow a stationary AR(1) process:

$$\ln \varepsilon_t^b = (1-\rho_b) \ln \varepsilon^b + \rho_b \ln \varepsilon_{t-1}^b + \sigma_b \eta_t^b; \eta_t^b \sim i.i.d.N(0;1) \tag{A.15}$$

$$\ln \varepsilon_t^l = (1-\rho_l) \ln \varepsilon^l + \rho_l \ln \varepsilon_{t-1}^l + \sigma_l \eta_t^l; \eta_t^l \sim i.i.d.N(0;1) \tag{A.16}$$

where $\rho_b \in (0; 1)$ and $\rho_l \in (0; 1)$ are the autoregressive parameters and $\sigma_b > 0$ and $\sigma_l > 0$ are the standard deviation of preference and labour supply shocks.

It is assumed that each household has access to the market of nominal bonds, $B_t(i)$, and participates in a state-contingent securities system that prevents idiosyncratic risks connected with wage rigidities. It also receives income from shares of intermediate-good firms, $A_t(i)$.[25] Thus, the intertemporal household's budget constraint is given by:

$$\frac{B_t(i)}{P_t R_t} + C_t(i) = \frac{B_{t-1}(i)}{P_t} + w_t(i) L_t(i) + A_t(i) \tag{A.17}$$

where R_t is the nominal interest rate.

The maximisation of the lifetime utility function (A.14), subject to a set of intertemporal budget constraints (A.17), results in the following Euler equation:

$$\frac{\varepsilon_t^b}{C_t(i)^{\delta_c}} = \beta E_t \left\{ \frac{\varepsilon_{t+1}^b}{C_{t+1}(i)^{\delta_c}} \frac{R_t}{\pi_{t+1}} \right\} \tag{A.18}$$

[25] The state-contingent securities system protects households from idiosyncratic risk arising from a staggered wage setting. It is assumed that payments from this system eliminate income inequalities between households in a given period.

Under the transversality condition, we have:

$$\lim_{t\to\infty}\beta^{t}\frac{\varepsilon_{t}^{b}}{C_{t}\left(i\right)^{\delta_{c}}}B_{t}\left(i\right)=0 \tag{A.19}$$

where $\pi_t = \dfrac{P_t}{P_{t-1}}$ is the inflation rate.

Similar to the intermediate firm's problem, each household chooses its wage, $w_t(i)$, according to the Calvo scheme (see Schmitt-Grohe, Uribe 2005). In every period, only a randomly chosen and constant part of households, $1-\theta_w \in (0;1)$, can reoptimise their wage. They maximise their lifetime utility, given by:

$$E_{t}\left\{\sum_{k=0}^{\infty}\beta^{k}\theta_{w}^{k}\varepsilon_{t+k}^{b}\left[\frac{C_{t+k}\left(i\right)^{1-\delta_{c}}}{1-\delta_{c}}-\varepsilon_{t+k}^{l}\frac{L_{t+k}\left(i\right)^{1+\delta_{l}}}{1+\delta_{l}}\right]\right\} \tag{A.20}$$

subject to the budget constraint (A.17) and labour demand (A.12). The first-order condition is given by:

$$E_{t}\left\{\sum_{k=0}^{\infty}\left(\beta\theta_{w}\right)^{k}L_{t+k}\left(i\right)^{*}\left[\left(1+\tau_{w}\right)MUL_{t+k}\left(i\right)^{*}-\frac{MUC_{t+k}\left(i\right)}{\prod_{m=1}^{k}\pi_{t+m}}w_{t}\left(i\right)^{*}\right]\right\} \tag{A.21}$$

where $MUL_t(i)^*$ is the optimal level of the marginal disutility of labour, $MUC_t(i)$ is the marginal utility of consumption and

$$L_{t+k}\left(i\right)^{*}=\left(\frac{w_{t}\left(i\right)^{*}}{w_{t+k}}\right)^{-\frac{1+\tau_{w}}{\tau_{w}}}L_{t+k} \tag{A.22}$$

where $w_t(i)^*$ is the optimal real wage.

Condition (A.21) indicates that a household chooses the optimal wage to equalise the average expected markup over the real marginal cost of working with the average expected marginal benefit of working, both expressed in utility terms (see Schmitt-Grohe and Uribe 2004). Moreover, under the assumption of symmetric equilibrium, all households that choose a wage in a given period choose the same real wage. The rest of the wages, namely $\theta_w \in (0;1)$, remain constant. The introduction of sticky wages means that the dynamics of the real wage (A.13) can be expressed as follows:

$$w_{t}=\left[\theta_{w}\left(\frac{w_{t-1}}{\pi_{t}}\right)^{-\frac{1}{\tau_{w}}}+\left(1-\theta_{w}\right)w_{t}^{*\,-\frac{1}{\tau_{w}}}\right]^{-\tau_{w}} \tag{A.23}$$

where w_t^* is the optimal real wage under symmetric equilibrium.

Finally, we impose the following equilibrium conditions on the goods and labour markets:

$$\frac{1}{\Delta_t(p)}\int_0^1 Y_t(j)\,dj = Y_t \tag{A.24}$$

$$\frac{1}{\Delta_t(w)}\int_0^1 L_t(i)\,di = L_t \tag{A.25}$$

and the aggregate demand equation:

$$Y_t = C_t \tag{A.26}$$

where $\Delta_t(p) = \int_0^1 \left(\frac{P_t(j)}{P_t}\right)^{-\frac{1+\tau_p}{\tau_p}} dj \geq 1$ and $\Delta_t(w) = \int_0^1 \left(\frac{w_t(i)}{w_t}\right)^{-\frac{1+\tau_w}{\tau_w}} dj \geq 1$ are inefficient price and wage dispersions, respectively.

Appendix B. Bayesian assessment of alternative policy rules

We empirically assess each rule $l = 1, 2, ..., 24$ by performing the Bayesian model comparison (see Kass and Raftery 1995; Fernandez-Villaverde and Rubio-Ramirez 2004, among others). This method is based on the marginal data density (MDD), which, for model M_l, is represented by:

$$p(\mathbf{Y_t} | \mathrm{M}_l) = \int_{\mathbf{\Theta}} p(\boldsymbol{\theta}, \boldsymbol{\phi}_l | \mathrm{M}_l) L(\mathbf{Y_t} | \boldsymbol{\theta}, \boldsymbol{\phi}_l, \boldsymbol{\omega}, \mathrm{M}_l) d\boldsymbol{\theta} d\boldsymbol{\phi}_\mathbf{l} \tag{B.1}$$

Each pair of models (M_l, M_k) can be compared by finding the posterior odds ratio, defined as:

$$POR_{l,k|\mathbf{Y_T}} = \frac{p(\mathrm{M}_l)}{p(\mathrm{M}_k)} \frac{p(\mathbf{Y_T} | \mathrm{M}_l)}{p(\mathbf{Y_T} | \mathrm{M}_k)} \tag{B.2}$$

where $\frac{p(\mathrm{M}_l)}{p(\mathrm{M}_k)}$ is the prior odds ratio. Under the assumption of equal prior model probabilities, the posterior odds ratio reduces to the Bayes factor, which, in log terms, is defined as:

$$\ln B_{l,k|\mathbf{Y_T}} = \ln p(\mathbf{Y_T} | \mathrm{M}_l) - \ln p(\mathbf{Y_T} | \mathrm{M}_k) \tag{B.4}$$

Kass and Raftery (1995) provide the decision rule based on the Bayes factor. According to them, a Bayes factor higher than 5 (in log terms) provides very strong evidence supporting model M_l against model M_k. Table B.1 contains the evaluation of the log of the MDD for all 24 estimated models, grouped by dynamic specifications. Performing a comparison of all the pairs of models shows that the model with forward-looking rule no. 5 is the most supported by the data.

Table B.1
The logarithms of marginal data density

Dynamic specification	**Rule number**							
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**
Backward-looking	-586.6	-592.7	-594.8	-600.9	-575.3	-581.7	-584.7	-591.3
Contemporaneous	-598.0	-603.8	-592.6	-596.8	-559.2	-565.4	-565.5	-571.7
Forward-looking	-576.7	-583.2	-581.7	-588.4	**-496.9**	-505.5	-505.7	-514.4

Notes: See Table 1. Bold represents the best model.
Source: own computations.

Appendix C. The importance of interest rate smoothing in optimised simple monetary policy rules

This appendix compares the welfare losses in Table 6 using the optimised simple rule approach (see Dennis 2004). Under this approach, we do not consider the whole posterior distribution of structural parameters and assume that vector **θ** is fixed on the posterior median. Table C.1 summarises the results. Contrary to the stochastic dominance approach, the optimised simple rule approach gives a different rule ranking with or without an interest rate smoothing mechanism. It shows that rule no. 1 generates lower welfare losses than rule no. 5.

Table C.1

The influence of interest rate smoothing on welfare loss – the optimised simple rule approach

Type of rule	Model 1		Model 2	
	No. of rule	**Welfare loss**	**No. of rule**	**Welfare loss**
Backward-looking (i = -1)	1	6.11	5	7.55
	2	1.34	6	1.05
	3	3.81	7	1.05
	4	2.84	8	1.03
Contemporaneous (i = 0)	1	5.72	5	6.98
	2	0.90	6	0.67
	3	1.43	7	0.62
	4	1.31	8	0.58
Forward-looking (i = 1)	1	2.59	5	7.21
	2	0.97	6	0.63
	3	1.57	7	0.83
	4	0.94	8	0.72

Notes:
Each welfare loss is calculated for the posterior median of structural parameters, all figures being multiplied by 10^3. The shaded cells show the model that generates the smallest welfare loss.

Jakie cechy optymalnych reguł polityki pieniężnej mają znaczenie dla gospodarki Polski? Analizy na podstawie relacji dominacji stochastycznej

Streszczenie

W niniejszej pracy zastosowano relacje dominacji stochastycznej do porównania optymalnych prostych reguł polityki pieniężnej w modelu racjonalnych oczekiwań z niepewnością parametrów. Wykorzystano nowy algorytm zaproponowany przez Górajskiego i Kuchtę (2021) do obliczania optymalnych rozkładów strat społecznych i optymalnych parametrów reakcji reguł polityki pieniężnej. Nasze podejście pozwala na wyznaczenie SDk-optymalnej reguły, która generuje najmniejszy rozkład start społecznych według relacji dominacji stochastycznej k-tego rzędu. W modelu Ercega, Hendersona i Levina (2000), oszacowanym dla gospodarki Polski na podstawie danych kwartalnych za lata 1995–2021, zbadano cechy charakterystyczne reguł polityki pieniężnej dla minimalizującego straty społeczne banku centralnego. W tym celu przy użyciu testów Lintona-Maasoumi-Whanga (2005) dla stochastycznej dominacji rzędu pierwszego oraz drugiego porównano rozkłady zminimalizowanych start dla alternatywnych specyfikacji optymalnych reguł polityki pieniężnej.

W empirycznym badaniu dla gospodarki Polski potwierdzono, że uwzględnienie przez decydentów mechanizmu wygładzania stóp procentowych i dodanie do optymalnych reguł polityki pieniężnej zmiennych opisujących aktywność gospodarczą lub poziom płac realnych pozwala na zmniejszenie strat społecznych. Wybór zmiennych reakcji do reguły dla optymalizującego banku centralnego zależy od dynamicznej specyfikacji reguły polityki i mechanizmu wygładzania stóp procentowych. Zidentyfikowano regułę SD1-optymalną dla Polski, minimalizującą funkcję strat społecznych, przy założeniu preferencji decydenta politycznego zgodnych z relacją dominacji stochastycznej rzędu pierwszego. Reguła SD1-optymalna reaguje na bieżące wartości inflacji, płac realnych oraz luki popytowej, a także uwzględnia mechanizm wygładzania stóp procentowych.

Słowa kluczowe: dominacja stochastyczna, optymalna polityka monetarna, rozkład strat w dobrobycie społecznym, model DSGE, niepewność parametrów