# Do Polish Consumers and Bank Analysts Learn How to Forecast Inflation?

## Czy polscy konsumenci i analitycy bankowi uczą się prognozować inflację?

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## Abstract

The aim of the paper is to test whether adaptive learning is a plausible way of describing the formation mechanism of inflation expectations of Polish consumers and bank analysts. I find some evidence that both groups of agents correct their expectations as new information is available. Moreover, it seems that consumers and bank analysts use rather simple forecasting rules and there are no large differences in the learning process between them.

**Keywords:** adaptive learning, inflation expectations

**JEL:** C53, D83, D84

## Streszczenie

Celem artykułu jest sprawdzenie, czy proces formułowania oczekiwań inflacyjnych przez polskich konsumentów i analityków bankowych można opisać za pomocą adaptacyjnego uczenia się. Wyniki badania sugerują, że obie grupy podmiotów korygują swoje oczekiwania, gdy nowe informacje stają się dostępne. Ponadto wydaje się, że konsumenci i analitycy bankowi używają raczej prostych reguł uczenia się i nie zachodzą znaczne różnice w procesach uczenia się między tymi dwoma grupami podmiotów.

**Słowa kluczowe:** procesy adaptacyjnego uczenia się, oczekiwania inflacyjne

# 1. Introduction

The rational expectations hypothesis puts strong informational requirements on economic agents. It is assumed that they know the actual structure of the economy and all its parameters, so forecast errors may result only from unpredictable random shocks. For many economists this assumption seems to be too restrictive and unrealistic. Therefore, recently an interest in developing expectations formation models, which depart from the perfect information assumption, is observed. In the last two decades learning has become one of the most popular approaches.

Writing about learning I refer to a concept of expectations formation in which economic agents revise their forecasting rules as new information appear. In the literature three types of learning might be distinguished: adaptive learning, eductive learning, and rational learning (Evans, Honkapohja 2001). In adaptive learning it is assumed that economic agents act as econometricians who estimate their forecasting rule every period. Eductive learning describes the process of reasoning about possible economic outcomes (in logical time) under the condition that other agents also make the inferring. The third concept, rational learning, refers to Bayesian estimation. It assumes that economic agents know the structural form of economy's model and have prior beliefs about parameter values. In this paper I will focus on adaptive learning approach only, as it is the most often met in the literature.

An important feature of the learning approach to modelling inflation expectations is that developments in the economy influence the way of forming expectations by agents (i.e. estimates of their forecasting rules), and the way economic agents form their expectations influences the outcomes of the economy. This aspect introduces additional dynamics to the economy. From a central bank point of view, it should be noted that assuming adaptive learning in expectations has implications for the conduct of monetary policy (for details see Evans, Honkapohja 2008).

In this paper I make use of the adaptive learning approach to analyse formation mechanism of Polish consumers' and bank analysts' inflation expectations. Previous studies indicated that consumers' inflation expectations do not fulfil unbiasedness and macroeconomic efficiency conditions (Łyziak 2005). However, in the long run consumers tend to adjust their expectations to the actual future inflation, which suggest that they are in some degree forward-looking (Łyziak, Stanisławska 2007). The properties of Polish commercial bank analysts' inflation expectations have not been examined in a rigorous way as far as I know. However, some preliminary evidence combined with studies on the forecasts of economists conducted in other countries (e.g. Baghestani, Kianian 1993; Lloyd Jr.

1999) suggests that this group of agents may also form their expectations in a manner inconsistent with the rational expectations hypothesis. This negative result on expectations' rationality was a motivation to test whether learning is a plausible way of describing their formation mechanism. More specifically, the paper tries to answer the following questions: which forecasting rule and learning algorithm economic agents under consideration use while forming inflation expectations? How fast is the process of learning? Are there any differences in the learning process between consumers and bank analysts? Do they optimally use available information?

The paper is organised as follows. Section 2 contains description of the considered learning schemes and the data on inflation expectations. Section 3 presents empirical results, while section 4 discusses the findings in light of previous studies. The final section concludes.

# 2. Description of survey data and method of analysis

Generally speaking, the procedure of testing adaptive learning as a possible mechanism of inflation expectations formation requires comparing number of series generated for various learning rules with various learning parameters to empirical measures of inflation expectations (Branch, Evans 2006; Pfajfar, Santoro 2006; Weber 2007). Based on assessment of fit of the simulated data to the actual ones, one can conclude about the plausible learning type, the forecasting rule and speed of learning.

Under adaptive learning economic agents use data available in period $t$ (denoted $x_t$) and a forecasting rule (called perceived law of motion, PLM)[1] with parameters estimated in the previous period ($\theta_{t-1}$) to formulate inflation forecasts for period $t+h$:

$$\pi_{t|t+h}^e = x_t \theta_{t-1} \qquad (1)$$

In the paper I consider six different PLMs that seem reasonable in forecasting inflation. The choice of PLMs is arbitral. I took under consideration very simple rules which do not put much informational requirements on economic agents, as well as more sophisticated ones.

PLM 1: $\pi_{t+h} = a_{1,t-1} + a_{2,t-1}\pi_{t-2} + \varepsilon_t$,

PLM 2: $\pi_{t+h} = a_{1,t-1} + a_{2,t-1}\pi_{t-2} + a_{3,t-1}y_{t-2} + \varepsilon_t$,

PLM 3: $\pi_{t+h} = a_{1,t-1} + a_{2,t-1}\pi_{t-2} + a_{3,t-1}e_{t-1} + \varepsilon_t$,

PLM 4: $\pi_{t+h} = a_{1,t-1} + a_{2,t-1}\pi_{t-2} + a_{3,t-1}i_{t-1} + \varepsilon_t$,

PLM 5: $\pi_{t+h} = a_{1,t-1} + a_{2,t-1}\pi_{t-2} + a_{3,t-1}\pi_{t+h}^{TAR} + \varepsilon_t$,

PLM 6: $\pi_{t+h} = a_{1,t-1} + a_{2,t-1}\pi_{t-2} + a_{3,t-1}\pi_{t-1}^{F} + \varepsilon_t$.

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[1] Perceived law of motion describes how economic agents interpret the inflation process. It might correspond with the true (structural) relation in the economy or not.

In the simplest case economic agents form their expectations only on the basis of past inflation data ($\pi_t$). Other forecasting rules include also the industrial production index ($y_t$), the real effective exchange rate ($e_t$) and the interest rate ($i_t$). It is also plausible that while forming their expectations they take into consideration the National Bank of Poland inflation target ($\pi_{t+h}^{TAR}$) and that consumers incorporate information on forecasts of professional economists[2] ($\pi_t^{F}$). I tested also other learning rules (containing food prices and bilateral exchange rates), but they did not perform well, so the results are not presented in the paper.

At the end of a period $t$, as new economic data become available, economic agents re-estimate their PLM. In general terms, the parameters' updating formula might be written in the following recursive form:

$$\theta_t = \theta_{t-1} + \gamma_t Q\left(t, \theta_{t-1}, x_t\right) \qquad (2)$$

where $\gamma_t$ denotes the "gain" sequence which determines how strongly parameters react to new data, and $Q(.)$ is a function describing the way in which the parameters are updated. Following Evans and Honkapohja (2001) one might distinguish two types of adaptive learning schemes in which agents correct for their past forecast errors: the least square learning and the stochastic gradient learning. The least square learning (LSL) scheme assumes that economic agents every period run a version of least square regression on their PLM. The stochastic gradient learning (SGL) constitutes a simple alternative to LSL. In this scheme agents also update their estimates according to past forecast errors, but they do not use information on variance–covariance matrix. In both learning schemes one might distinguish a decreasing gain ($\gamma_t = \alpha t^{-1}$) and a constant gain ($\gamma_t = \gamma$) case.[3]

Additionally, I refer to a modification of LSL and SGL schemes proposed by Pfajfar and Santoro (2006). They assume that agents update their PLM according to new information about future inflation, so function $Q(.)$ includes "future errors".[4] This version of learning, in contrast to the previous one, presumes more forward–lookingness of economic agents. Further details on all learning algorithms applied in the paper are given in section 3.

The testing procedure is as follows. First, for a given learning scheme, series of inflation forecasts are generated according to (1) and (2). Then I search for the value of $\gamma$ parameter that minimises the mean square error in the sample, defined as difference between the generated and the actual data on inflation expectations. In the case of constant gain scheme, values of $\gamma$ parameter

range from 0 to 1, with distance between parameters equal to 0.001. For the decreasing gain scheme ($\gamma_t = \alpha t^{-1}$), series are computed for values of $\alpha$ varying from 0 to 1, also with distance equal to 0.001.[5] The initial values of the recursive algorithm are set equal to estimation results of PLMs for the pre-forecasting period (01.1996 to 12.1999).

Due to the risk of over-fitting it would be advisable to choose best fitting $\gamma$ parameters and to assess the performance of PLMs with these parameters on separate samples as done in Branch and Evans (2006) and Weber (2007). However, due to limitations in sample length I cannot afford to split it in two.

As a reference series for inflation expectations, survey measures are employed: one derived from consumer survey carried by Ipsos, and the second based on Reuters pool of bank analysts.[6] In the consumer survey respondents declare the expected direction and the intensity of price changes during the next 12 months, without providing exact numbers. The survey question is formulated in the following way: „*By comparison with the past 12 months, how do you expect that consumer prices will develop in the next 12 months? They will: (a1) increase more rapidly; (a2) increase at the same rate; (a3) increase at a slower rate; (b) stay about the same; (c) fall; (d) don't know*". These qualitative answers are then quantified according to the probability procedure (for details see: Łyziak 2005). The second survey includes a question on the expected yearly inflation in the following 11 months.[7] Its main shortcoming is the small number of respondents which varies from 7 to 29. Both survey series together with the current inflation rate are presented in Figure 1.

Despite the fact that both surveys have long history (Ipsos pool started in 1992 and Reuters in 1996), a much shorter sample, starting in 01.2000, is employed in the analysis. The main reason is that in the nineties Poland experienced high and volatile inflation rates and structural changes, including a change in the monetary regime. In contrast, the post 1999 period is characterised by relatively low, stable inflation and homogeneous monetary policy regime. Moreover, I have to allow some pre-forecasting period to set initial values of parameters in PLMs. The analysis was conducted also on a shorter sample starting in 01.2002, when the disinflation process was terminated, but the conclusions were the same.

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[2] I.e. forecasts of bank analysts from Reuters pool.

[3] The type of "gain" parameter affects convergence of learning process to rational expectations. Generally speaking, in the case of decreasing "gain" the process converges, and in the case of constant "gain" it does not converge (Evans, Honkapohja 2001).

[4] They borrowed this concept from Fuchs (1979).

[5] The choice of parameter value range and density of search was based on results of Pfajfar and Santoro (2006) and Weber (2007).

[6] For description of measurement methods of inflation expectations in general see, for example, Pesaran (1989), and specifically on Polish data see: Łyziak, Stanisławska (2006).
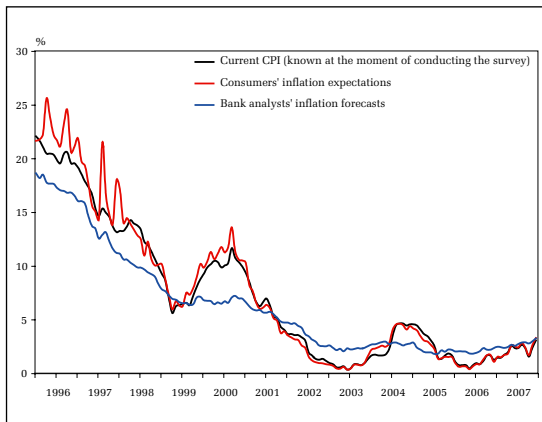
[7] Before November 2001 the forecasting horizon was equal to 12 months, but in the analysis I adopt homogenous forecast horizon of 11 months in the whole sample.

**Figure 1.** *Current inflation and inflation expectations in Poland, 1996–2007*



*Source: GUS, Ipsos, Reuters data and own calculations.*

## 3. Empirical results on adaptive learning

This section presents empirical results on the ability of various adaptive learning processes to match survey data on inflation expectations of Polish consumers and bank analysts. Firstly, I focus on least square learning, in which economic agents correct forecasts according to past forecast errors, and secondly on a scheme in which they correct them for new available information on future inflation.

### 3.1. Learning with regard to past forecast errors

The updating formula of PLM's parameters under least square learning might be written in a recursive form as follows:

$$\theta_t = \theta_{t-1} + \gamma_t R_{t-1}^{-1} x_{t-h-1}' \left( \pi_{t-1} - x_{t-h-1} \theta_{t-h-2} \right)$$
$$R_t = R_{t-1} + \gamma_t \left( x_{t-h-1}' x_{t-h-1} - R_{t-1} \right) \tag{3}$$

where $R_t$ denotes variance-covariance matrix, and $h$ equals 11 for bank analysts and 12 for consumers. If constant gain scheme is assumed, then $\gamma_t$ is set equal to a constant. It means that past observations are geometrically discounted (Sargent 1999), so this

learning pattern is more robust to structural changes. If one assumes decreasing gain scheme in which gain parameter is determined as $\gamma_t = \alpha t^{-1}$ ($\alpha > 0$) (3) becomes a version of weighted least squares. If $\alpha$ is set equal to 1, then (3) is equivalent to recursive ordinary least squares (Basdevant 2003).

Results presented in Table 1 support the hypothesis that agents, both consumers and bank analysts, learn from their past mistakes to formulate better forecasts of future inflation. Values of "gain" parameters which fit best survey data (in MSE terms), without regard of learning scheme assumed, differ from zero, but are rather small. It seems that the learning process describes the behaviour of bank analysts better, as MSE statistics for this group of agents are significantly smaller than for consumers. As one compares two learning types: constant gain and decreasing gain, it turns out that for most of PLMs better fit is obtained in the case of the latter. It means that the process of forming inflation expectations by Polish economic agents might converge to rational expectations.

It is a little bit surprising that there is so little difference in the learning process between consumers and bank analysts. Both groups of agents seem to formulate expectations based on simple forecasting rules, PLM 1 or PLM 4, which include only the last known inflation rate or the last known inflation and interest rate. The worst performing forecast rule is one containing the real exchange rate (PLM 3). Also a weak performance of PLMs using information concerning the National Bank of Poland's inflation target is somewhat surprising. The reason for which consumers prefer these two simple forecasting rules might be that data on current inflation and interest rates are easily observable and relatively well understood. All data included in the PLMs under consideration are published by the Central Statistical Office (GUS) or the NBP, but only changes in prices and interest rates are directly observable by consumers in their everyday life (while doing shopping or looking at credit costs). Therefore, the cost of collecting and processing these data might be lower. It is more difficult to explain why bank analysts do not make use of more information to forecast future inflation as in their case accessibility to statistical data is not a problem. It is possible that they put more weight on quarterly data
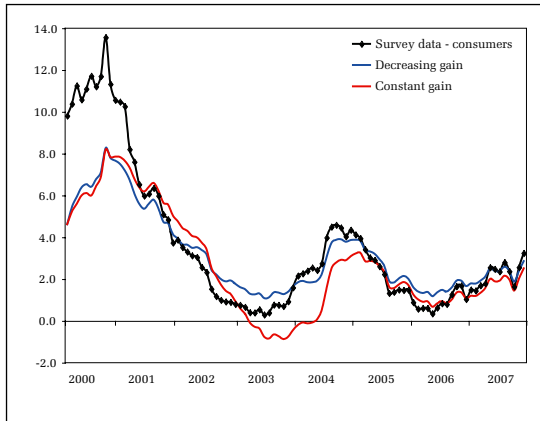
**Table 1.** *Best-fitting gain parameters and MSE (in brackets)[a] for least square learning, correction of past errors*

| | Consumers | | | | Bank analysts | | | |
|---|---|---|---|---|---|---|---|---|
| | constant gain | | decreasing gain | | constant gain | | decreasing gain | |
| PLM 1 | 0.006 | (3.54) | 0.031· t$^{-1}$ | (2.73) | 0.007 | (2.34) | 0.039· t$^{-1}$ | (0.53) |
| PLM 2 | 0.005 | (5.07) | 0.036· t$^{-1}$ | (4.28) | 0.006 | (2.57) | 0.049· t$^{-1}$ | (1.14) |
| PLM 3 | 0.005 | (14.23) | 0.041· t$^{-1}$ | (18.74) | 0.010 | (10.23) | 0.059· t$^{-1}$ | (17.81) |
| PLM 4 | 0.006 | (3.34) | 0.035· t$^{-1}$ | (2.89) | 0.007 | (2.06) | 0.046· t$^{-1}$ | (0.63) |
| PLM 5 | 0.001 | (13.66) | 0.006· t$^{-1}$ | (13.94) | 0.003 | (5.06) | 0.015· t$^{-1}$ | (6.08) |
| PLM 6 | 0.001 | (11.81) | 0.006· t$^{-1}$ | (11.71) | - | - | - | - |

[a]   $MSE = \frac{1}{n}\sum_{i=1}^{n}\left( \hat{\pi}_{t+h} - \pi_{t|t+h}^e \right)^2$, where: $\hat{\pi}_{t+h}$ denotes series generated according to adaptive learning scheme with given gain parameter value, and $\pi_{t|t+h}^e$ denotes survey data.
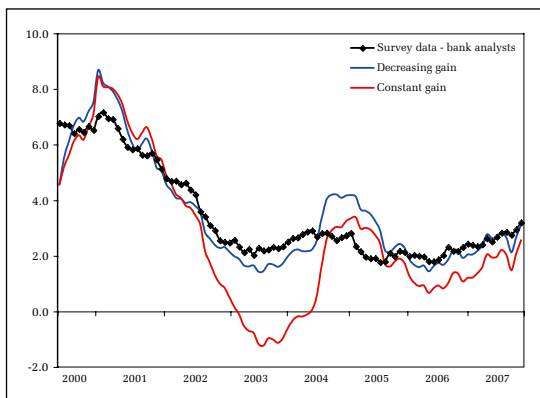
*Source: own calculations based on Ipsos, Reuters and GUS data.*

Figure 2. *Survey data on consumers' inflation expectations and data generated according to PLM 1 with best-fitting gain parameters*



*Source: own calculations based on GUS and Ipsos data.*

Figure 3. *Survey data on bank analysts' inflation expectations and data generated according to PLM 1 with best-fitting gain parameters*



*Source: own calculations based on GUS and Reuters data.*

(like GDP figures necessary to estimate output gap) than on monthly indicators. Weak performance of PLM 5 might be accounted for by the low credibility of the NBP among consumers and the history of frequent missing of the target (Łyziak et al. 2007).

Consumers and bank analysts do not differ much in the value of the "gain" parameter: the gain seems to be only slightly higher for the latter. However comparing them is not straightforward due to the difference in the forecasting horizon.

As the results for a simpler learning method, the stochastic gradient, are very similar (see Table in Annex), they are not discussed in detail. The only thing worth noticing is that the stochastic gradient scheme seems to match survey data to a lesser extent that the least square learning. This would suggest that economic agents under consideration use relatively more complicated, closer to rational expectations, ways to form their expectations.

### 3.2. Learning with regard to "future forecast errors"

Another learning scheme tested in the paper assumes that agents correct their forecasts not with regard to past errors, but to newly appearing information about future inflation. The updating formula for PLMs parameters in this case is as follows:

$$
\theta_t = \theta_{t-1} + \gamma_t R_{t-1}^{-1} x_t' \left( \pi_{t+h} - x_t \theta_{t-1} \right)
$$
$$
R_t = R_{t-1} + \gamma_t \left( x_t' x_t - R_{t-1} \right) \tag{4}
$$

Also in this case I have found evidence in favour of learning in the expectations formation process for both groups of agents (Table 2). It means that they use information about future inflation to improve their forecasts. It is worth noticing that values of "gain" parameters are significantly higher than in the case of the learning mechanism described in the previous sub-section, but the fit to survey data is generally worse. The decreasing gain scheme seems to outperform slightly the constant gain scheme.

Looking at various forecasting rules, it turns out again that the simplest perceived law of motion (PLM 1) performs relatively well in comparison to more sophisticated ones. Results obtained under this learning type are a little bit more supportive – in comparison to the previous ones – to hypothesis that consumers forming expectations take into consideration professional
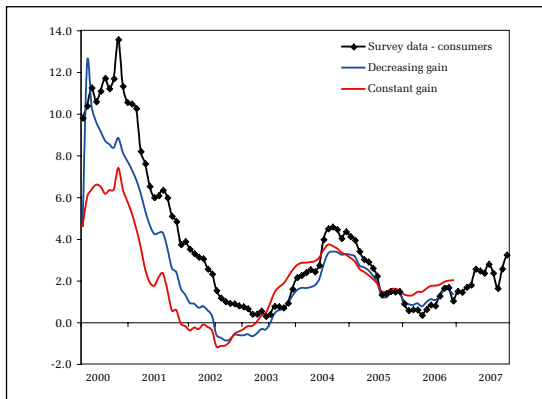
Table 2. *Best-fitting gain parameters and MSE (in brackets)[a] for least square learning, correction of "future errors"*

| | Consumers | | | Bank analysts | | |
|---|---|---|---|---|---|---|
| | constant gain | | decreasing gain | | constant gain | | decreasing gain | |

| | constant gain | decreasing gain | | constant gain | decreasing gain | |
|---|---|---|---|---|---|---|
| PLM 1 | 0.055 (7.59) | 0.489· t$^{-1}$ | (2.69) | 0.216 (3.92) | 0.226· t$^{-1}$ | (3.28) |
| PLM 2 | 0.012 (8.27) | 0.188· t$^{-1}$ | (5.48) | 0.150 (4.71) | 0.140· t$^{-1}$ | (5.29) |
| PLM 3 | 0.160 (10.80) | 0.169· t$^{-1}$ | (10.86) | 0.132 (5.01) | 0.170· t$^{-1}$ | (7.78) |
| PLM 4 | 0.046 (7.62) | 0.289· t$^{-1}$ | (3.43) | 0.187 (4.23) | 0.193· t$^{-1}$ | (3.38) |
| PLM 5 | 0.085 (8.61) | 0.226· t$^{-1}$ | (5.81) | 0.072 (3.36) | 0.150· t$^{-1}$ | (0.48) |
| PLM 6 | 0.060 (8.11) | 0.238· t$^{-1}$ | (3.91) | - | - | - |

a  $MSE = \frac{1}{n}\sum_{i=1}^{n}\left( \hat{\pi}_{t+h} - \pi_{t|t+h}^e \right)^2$, where: $\hat{\pi}_{t+h}$ denotes series generated according to adaptive learning scheme with given gain parameter value, and $\pi_{t|t+h}^e$ denotes survey data.
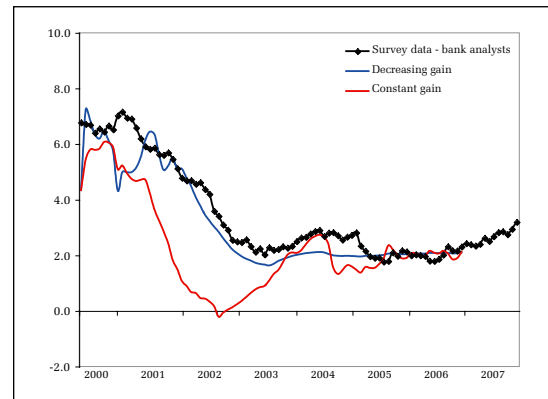
*Source: own calculations based on Ipsos, Reuters and GUS data.*

Figure 4. *Survey data on consumers' inflation expectations and data generated according to PLM 1 with best-fitting gain parameters*



*Source: own calculations based on GUS and Ipsos data.*

Figure 5. *Survey data on bank analysts' inflation expectations and data generated according to PLM 5 with best-fitting gain parameters*



*Source: own calculations based on GUS and Reuters data.*

forecasters' opinion about future price developments. In the case of bank analysts, good performance of PLM 5, including the last known inflation rate and the inflation target, is worth noticing. It seems that inflation forecasts of this group of agents are quite anchored and that bank analysts efficiently use information about future inflation changes.

The results for stochastic gradient learning are quite similar, but the fit to survey data is much weaker (see Table 5 in the Annex).

### 3.3. Discussion of results

The results obtained for Polish consumers and bank analysts are quite in line with results of similar studies conducted on US and euro area data. Pfajfar and Santoro (2006) on the basis of the Michigan Survey find that a significant part of consumers tend to adaptively "learn" while forming inflation expectations.[8] The authors do not find large differences in fit between least square and stochastic gradient learning or constant and decreasing gain schemes. Moreover, estimated gain parameter values are rather small. Also Weber (2007) finds out that inflation expectations of consumers and professional forecasters in several euro area countries are formed in a way consistent with adaptive learning model. She noticed that simple forecasting rules, like AR(1) inflation process, in general perform better than more sophisticated PLMs. Moreover, she finds some evidence in favour of constant gain scheme[9], and of higher values of the gain parameter in countries characterised by higher inflation rates. Comparing the learning process

between consumers and professional forecasters, her results suggest that the latter use shorter period of past data to forecast inflation.

The evidence regarding learning in inflation expectations formation confirms the previous findings of Łyziak and Stanisławska (2006) that Polish consumers' inflation expectations are not purely backward-looking. They attempt to improve forecasts of future inflation, and probably use not only information on past inflation, but also on interest rates. Moreover, consumers showed significant forward-lookingness in 2004 in predicting the increase of inflation after Poland's accession to the European Union, as suggested by high gains and good fit of the PLM 1 in "future errors" learning scheme. It is a little bit surprising that there are no significant differences in learning speed between consumers and bank analysts. The distinction between these two groups manifest itself in the efficiency of use of information on future price changes and central bank's inflation target (recall the very good performance of PLM 5 in section 3.2) rather than in the speed of learning.

It is also possible to test whether Polish consumers and bank analysts used an optimal gain – i.e. gain associated with the minimum MSE of forecasts of future inflation – in their learning processes. Branch and Evans (2006) and Weber (2007) find that professional forecasters and consumers do not employ optimal gain while forming inflation expectations. Similar results are obtained for Polish data. Figure 6 and Figure 7 plot the MSE statistics for inflation forecast and for deviations from survey data, as a function of gain parameter[10], for

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[8] They found significant heterogeneity in consumers' behaviour. Agents whose expectations are placed on the right hand side of distribution, tend to formulate forecasts according to adaptive learning scheme. On the contrary, the rest of agents use rather static expectations.

[9] Although, in her version of RLS with decreasing gain α was restricted to 1.

[10] Optimal gain might be different for consumers and bank analysts due to the differences in their forecasting horizon. Best-fitting gains on Figure 6 and Figure 7 might slightly differ from ones presented in the previous section, as they are set on shorter sample (ending in 2006, not 2007) to assure comparability with optimal gain.

**Table 3.** *Comparison of inflation forecasts accuracy between series generated according to adaptive learning processes with best fitting and optimal gains[a]*

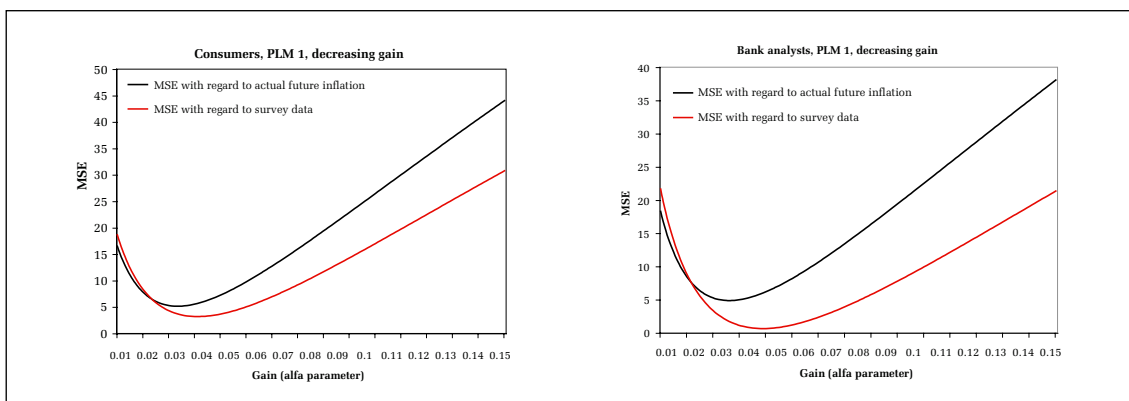| | Consumers | | Bank analysts | |
|---|---|---|---|---|
| | Forecast RMSE for optimal gain[b] | Forecast RMSE for best fitting gain[c] | Forecast RMSE for optimal gain[b] | Forecast RMSE for best fitting gain[c] |
| learning with regard to past errors | | | | |
| Decreasing gain | 2.25 | 2.37 | 2.19 | 2.44 |
| Constant gain | 2.54 | 2.96 | 2.51 | 2.93 |
| learning with regard to "future errors" | | | | |
| Decreasing gain | 2.33 | 2.42 | 1.49 | 1.89 |
| Constant gain | 0.93 | 1.63 | 0.79 | 1.13 |

[a] All but one statistics refer to PLM 1. The exception is learning with regard to future errors for bank analyst, for which statistics refer to PLM 5.

[b] $RMSE = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(\hat{\pi}_{t+h} - \pi_{t+h})^2}$, where $\hat{\pi}_{t+h}$ denotes series generated according to adaptive learning scheme with optimal gain, and $\pi_{t+h}$ denotes the actual future inflation.

[c] $RMSE = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(\hat{\pi}^e_{t+h} - \pi_{t+h})^2}$ where $\hat{\pi}^e_{t+h}$ denotes series generated according to adaptive learning scheme with best fitting gain, and $\pi_{t+h}$ denotes the actual future inflation.

*Source: own calculations based on GUS, Ipsos and Reuters data.*

**Figure 6.** *Mean square forecast errors[a] and mean square deviations from survey data[b] as a function of gain parameter - PLM 1, decreasing gain, learning with regard to past errors*



[a] $MSE = \frac{1}{n}\sum_{i=1}^{n}(\hat{\pi}_{t+h} - \pi_{t+h})^2$, where: $\hat{\pi}_{t+h}$ denotes series generated according to adaptive learning scheme with given gain parameter value, and $\pi_{t+h}$ denotes the actual future inflation.

[b] $MSE = \frac{1}{n}\sum_{i=1}^{n}(\hat{\pi}_{t+h} - \pi^e_{t|t+h})^2$, where: $\hat{\pi}_{t+h}$ denotes series generated according to adaptive learning scheme with given gain parameter value, and $\pi^e_{t|t+h}$ denotes the survey data.

*Source: own calculations based on GUS, Ipsos and Reuters data.*

**Figure 7.** *Mean square forecast errors[a] and mean square deviations from survey data[b] as a function of gain parameter - PLM 1, decreasing gain, learning with regard to "future errors"*
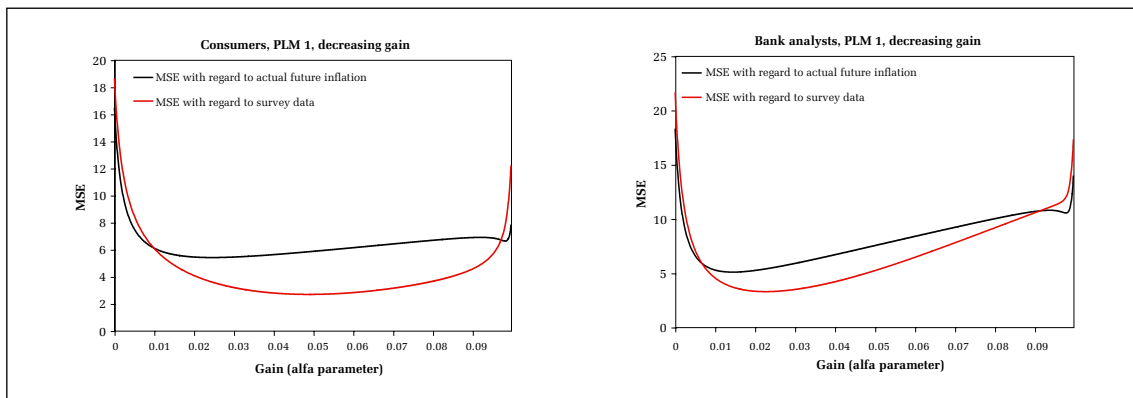


[a] $MSE = \frac{1}{n}\sum_{i=1}^{n}(\hat{\pi}_{t+h} - \pi_{t+h})^2$, where: $\hat{\pi}_{t+h}$ denotes series generated according to adaptive learning scheme with given gain parameter value, and $\pi_{t+h}$ denotes the actual future inflation.

[b] $MSE = \frac{1}{n}\sum_{i=1}^{n}(\hat{\pi}_{t+h} - \pi^e_{t|t+h})^2$, where: $\hat{\pi}_{t+h}$ denotes series generated according to adaptive learning scheme with given gain parameter value, and $\pi^e_{t|t+h}$ denotes the survey data.

*Source: own calculations based on GUS, Ipsos and Reuters data.*

selected learning schemes[11]. It is clearly seen that values of the best fitting survey data gain (i.e. minimising the MSE calculated with respect to survey data) and optimal gain (i.e. minimising the MSE calculated with respect to future actual inflation) are close to each other, but different. It means that both groups of agents could slightly improve their forecasts' accuracy within adaptive learning algorithm (Table 3).

## 4. Conclusions

The results of this study suggest that the formation process of inflation expectations of Polish consumers and bank analysts might be described by adaptive learning. Both groups of agents correct forecasts in accordance with past errors, and with new information about future inflation. I find some evidence in favour of least square learning and decreasing gain as compared to stochastic gradient learning and constant gain scheme. Nevertheless, it is possible that other kind of learning, i.e. eductive or rational, takes place, but it wasn't tested.

The learning algorithm is vastly important in the modelling of expectations. However, it also gives information about the processing skills of economic agents and the amount of information they use while forming forecasts. It seems that Polish economic agents use a relatively more complex way of forming expectations and their forecasts might converge to rational expectations.

If one considers adaptive learning in its traditional meaning, i.e. as correcting for past errors, it turns out that economic agents while formulating their expectations employ rather simple forecasting rules, which include the past inflation rate or the past inflation rate and the interest rate. There are no large differences between consumers and bank analysts in the learning process with regard to speed of learning or variables taken under consideration in formulating forecasts. However, the generated series imitate the behaviour of bank analysts more precisely than that of consumers.

If one allows consumers and bank analysts to correct their predictions as new information about future inflation appears, i.e. to revise forecasts with regard to "future errors", the difference between these two groups becomes more apparent. Bank analysts seem to employ information on the central bank's inflation target and future price changes to a  greater degree than consumers.

Finally, both groups of agents could slightly improve the accuracy of their forecasts within the given adaptive learning scheme, as best fitting gains differ from optimal gains.

The caveat is that due to a relatively short sample both: the choice of parameters' values and the assessment of the performance of competitive PLMs and learning types is done on the same sample. Therefore, as more data is gathered, it would be advisable to repeat this analysis for two periods: "in-sample" and "out-of-sample".

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[11]  Results for other learning schemes are very similar, so are not presented here.

## Annex. Results for Stochastic Gradient Learning

Under stochastic gradient learning economic agents do not take account of second moments of independent variables in PLM, so the recursive algorithm of parameters' updating might be written as follows:

- for learning with regard to past forecast errors:

$$\theta_t = \theta_{t-1} + \gamma_t x_{t-h-1}^{'} \left( \pi_{t-1} - x_{t-h-1}\theta_{t-h-2} \right) \qquad (5)$$

- for learning with regard to "future forecast errors":

$$\theta_t = \theta_{t-1} + \gamma_t x_t^{'} \left( \pi_{t+h} - x_t\theta_{t-1} \right) \qquad (6)$$

Table 4 and Table 5 present results obtained for both types of stochastic gradient learning.

Table 4. *Best-fit gain parameters and MSE (in brackets) for stochastic gradient learning, basic scheme (correction of past errors)*

| | Consumers | | | | Bank analysts | | | |
|---|---|---|---|---|---|---|---|---|
| | constant gain | | decreasing gain | | constant gain | | decreasing gain | |
| PLM 1 | 0.001 | (12.07) | $0.007 \cdot t^{-1}$ | (7.54) | 0.001 | (16.22) | $0.005 \cdot t^{-1}$ | (13.62) |
| PLM 2 | 0.001 | (17.19) | $0.006 \cdot t^{-1}$ | (7.76) | 0.001 | (17.47) | $0.005 \cdot t^{-1}$ | (11.34) |
| PLM 3 | 0.001 | (10.44) | $0.008 \cdot t^{-1}$ | (9.84) | 0.001 | (14.25) | $0.007 \cdot t^{-1}$ | (16.95) |
| PLM 4 | 0.001 | (12.13) | $0.009 \cdot t^{-1}$ | (8.01) | 0.001 | (15.92) | $0.006 \cdot t^{-1}$ | (13.52) |
| PLM 5 | 0.001 | (7.05) | $0.007 \cdot t^{-1}$ | (2.77) | 0.001 | (3.45) | $0.005 \cdot t^{-1}$ | (1.06) |
| PLM 6 | 0.001 | (8.66) | $0.004 \cdot t^{-1}$ | (4.85) | - | - | - | - |

*Source: own calculations based on Ipsos, Reuters and GUS data.*

Table 5. *Best-fit gain parameters and MSE (in brackets) for stochastic gradient learning, extended scheme (correction of future errors)*

| | Consumers | | | | Bank analysts | | | |
|---|---|---|---|---|---|---|---|---|
| | constant gain | | decreasing gain | | constant gain | | decreasing gain | |
| PLM 1 | 0.015 | (11.74) | $0.005\ t^{-1}$ | (16.28) | 0.019 | (10.84) | $0.004\ t^{-1}$ | (17.85) |
| PLM 2 | 0.001 | (10.45) | $0.003\ t^{-1}$ | (11.43) | 0.004 | (8.89) | $0.003\ t^{-1}$ | (12.71) |
| PLM 3 | 0.010 | (12.43) | $0.031\ t^{-1}$ | (17.04) | 0.012 | (7.08) | $0.033\ t^{-1}$ | (16.52) |
| PLM 4 | 0.015 | (11.67) | $0.005\ t^{-1}$ | (16.24) | 0.019 | (10.74) | $0.004\ t^{-1}$ | (17.81) |
| PLM 5 | 0.007 | (8.69) | $0.008\ t^{-1}$ | (8.81) | 0.008 | (3.99) | $0.006\ t^{-1}$ | (4.31) |
| PLM 6 | 0.006 | (8.50) | $0.007\ t^{-1}$ | (6.96) | - | - | - | - |

*Source: own calculations based on Ipsos, Reuters and GUS data.*

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