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# Bankrupt UK cities: PD model for credit risk in sub-sovereign sector

Lukasz Prorokowski*

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## Abstract

We develop a PD model (PD – probability of default) for sub-sovereign entities, namely UK municipalities. Our methodology serves as an alternative for banks that use the standardised approach or scorecard-based models for assessing the probability of default for municipalities, local authorities and other sub-sovereign entities.

Focusing on credit exposures to municipalities, we address the concerns that sub-sovereign and sovereign entities are nowadays more risky than large corporate or bank entities. Furthermore, discussing the current and forthcoming regulatory frameworks for credit risk models, we point to the existence of contradictory regulations and argue that dispensing with the conservative approach may lead to a build-up of credit risk that cannot be accurately captured. With this in mind, we argue that PD models should remain conservative so that banks can accumulate sufficient capital to cover the crisis-induced default exposures.

**Keywords:** credit risk, probability of default (PD model), local authorities, capital requirements regulation (CRR), IFRS 9

**JEL:** R5, H7, G21, G28

* H.L. Prorokowski LLC; e-mail: lukas.prorokowski@gmail.com.

# 1. Introduction

## 1.1. Purpose of this study

In light of the recent wave of sovereign bankruptcies, especially evident across US municipalities and local authorities, as well as growing concerns regarding the UK government's ability to support its entities (see Medioli, Van Praagh, Tudela 2014; Tudela, Medioli, Van Praagh 2013), we have developed a PD model (PD = probability of default) for UK municipalities. This model serves to assess the general risk of UK cities and UK government supported entities defaulting on their credit obligations. The model is useful for the credit risk departments of commercial banks and other financial services firms that have exposures to sovereign and sub-sovereign entities. Considering the above characteristics, the methodology presented in this paper should serve as an alternative for banks that use the standardised approach in credit risk or scorecard-based PD models for assessing the probability of default for municipalities, local authorities and other sub-sovereign entities. By introducing the statistically based probit model, we expect that banks benefit from a reduction in overrides made by credit officers to the existing scorecard-based gradings.

The findings presented in this paper serve to reconcile the concerns that sub-sovereign and sovereign entities are nowadays more risky than large corporate or bank entities. This is especially important when creating low default portfolios (LDP) that include local authorities and municipalities, as strict CRR rules apply to the concept of LDPs. As it transpires, 38% of the LDPs among European banks consist of sovereign and sub-sovereign exposures (EBA 2015).

Modelling the probability of default for entities included in LDPs is problematic given the limited historical bankruptcies and the lack of up-to-date external agency ratings for UK municipalities and local authorities (Standard & Poor's 2013). At this point, we propose a PD model that can be rooted in the internal grading replication approaches by estimating the model parameters based on the least squares fit to any internal grading process. With this in mind, we propose to transform internal gradings into the distance-to-default measures.

Highlighting the core purpose of this study, one should note that this paper attempts to present an innovative approach to credit risk modelling with a focus on modelling probabilities of default for UK municipalities. The aim of this paper is to provide a flexible modelling basis that can be utilised and modified to the needs of prospective users. At this point, the model can be applied in its current form or undergo further modifications to meet the new requirements. We also assume that only certain parts of the methodology can be found useful for prospective users in their efforts to develop similar models. We view this paper as a source of inspiration for other researchers, analysts and practitioners.

## 1.2. Study background

Although bankruptcies among municipalities remain rare, many local authorities are faced with funding cutbacks, decreased ability to generate income from taxes and a growing demand for social spending, as highlighted in the recent reports of Governing Institute (2015) and Moore (2013). Against this backdrop, years 2013–2015 saw an increase in the number of cities declaring bankruptcy. Although the majority of defaults are taking place in the United States, the recent bankruptcies of Detroit, Stockton and Jefferson County sent a clear message for municipal bondholders, central banks and regulators in Europe that regional governments struggle to pay off debts during the global economic downturn and the ensuing austerity (see United Cities and Local Governments 2014). Furthermore, the UK media report that cities in Northern England (e.g. Grimsby, Blackpool, Stroke-on-Trent, Hull and Burnley) are struggling economically given the steep decline in the UK's economy (Brown 2016).

Given the increased default rates among municipalities and local authorities, modelling credit risk exposures to these entities becomes important for the commercial credit institutions and regulators. Banks that have exposures to municipalities should develop appropriate models assessing credit risk exposures in this sector in order to effectively allocate loss capital. Regulators should have a PD model for municipalities to effectively assess and compare credit risk capital charges calculated by the banks for the credit portfolio of sovereign and sub-sovereign asset classes.

Addressing the aforementioned needs of different institutions we propose a new PD model for assessing credit risk of municipalities. Since our PD model is estimated in the distance-to-default space, we note that banks have developed an array of different methodologies for estimating the probability of default of their debtors. Among these approaches to modelling credit risk, the distance--to-default measures gained some traction in the banking industry (Gornall, Strebulaev 2015). However, the increased regulatory scrutiny makes it challenging to apply the distance-to-default methodologies to a process of assessing default probabilities in certain industries. For example, Harada and Ito (2010), as well as Chan-Lau and Sy (2006) argue that PD models based on the distance-to-default measures are not suitable for predicting episodes of distress when applied to the banking industry.

While acknowledging that the distance-to-default measures should be used with caution when applied to the financial industry debtors, Zielinski (2013) and Harada and Ito (2010) point to the fact that these measures may underestimate the probability of a financial institution being forced by regulators to take some corrective steps. As it transpires, the authorities would take a number of statutory actions to avoid large costs associated with banks' defaults (Prorokowski 2011). However, we argue that municipalities and local authorities differ from other debtors, because they do not have complex debt structures and do not operate in a similarly challenging/competitive or closely scrutinised environment. Thus, the distance-to-default measures can be applicable to sub-sovereign entities. Furthermore, in light of the arguments by Nagel and Purnanandam (2015) that the distance--to-default measures may underestimate the probability of default, we propose solutions that ensure the conservatism of the PD model. Finally, we note that financial authorities have started to use the distance-to-default measures to monitor systemic risk (Blancher et al. 2013; Saldias 2012). The European Central Bank regards this indicator as a forward-looking measure providing an early warning signal of financial instability – ECB (2005); and De Nicolo and Tieman (2006) found the PD models based on the distance-to-default measures to be comprehensive indicators of credit risk.

As shown in Table 1, there is an array of credit risk models suitable for PD modelling, starting with the Z-model developed by Altman (1968) and ending with the contingent claims approach modelling propagated by Gray and Malone (2008) and the convulsion PD model developed by Iqbal and Ali (2012) for the internal risk based (IRB) approach. These modelling methodologies can be divided into the structural models and the intensity-based approaches. There are also well-established regulatory frameworks for calculating credit risk capital charges. The regulatory standards for the standardised approach in credit risk are currently undergoing their second revision (BCBS 2015b). The two classic structural approaches to PD modelling are represented by the Merton model (Merton 1974) and the

Black and Cox's first-passage-time model (Black, Cox 1976). At a later stage, the Black-Scholes-Merton framework was extended to build models that include the empirical distribution of distance-to-default (Crosbie, Bohn 2003; Kealhofer 2003a, 2003b; Vasicek 1984). The advantage of using the distance--to-default models is the fact that the probability of default can be derived directly from the known distribution of assets or from the established default rate for a given distance-to-default level (Crosbie, Bohn 2003). Furthermore, the majority of credit risk modellers use the conditional probability of default models – logit and probit (Marin, Ponce 2005). The conditional probability models return the probability that a particular default event belongs to a certain group of observations, once the values of the independent variables for that default event are known. Logit models became increasingly popular among practitioners, for instance, the RiskCalc developed by Moody's in 2000 is based on a logit model calculating default probabilities for companies not listed on stock exchanges. Carey and Hrycay (2001) as well as Westgaard and Van der Wijst (2001) also embark on logit models and financial ratios to calculate probabilities of default.

Recently, scholars have started to explore the applications of the contingent claims approach (CCA) in the distance-to-default space in order to assess credit risk. This new research field extends the use of structural models with the add-ons of macro-finance models integrating traditional approaches (e.g. Merton 1974) with modern option pricing modelling frameworks (e.g. Gray, Malone 2008). According to Aktug (2014), the CCA addresses the flaws of the structural models that underestimate the true risk measures. Kozak, Aaron and Gauthier (2005) explains that the CCA is built on balance sheet data (historical data) as well as equity market information (forward looking data) to derive the distance-to--default measures. At this point, Lewis (2012) found the combination of the balance sheet data with the high frequency price information from equity markets to be a good forward looking indicator of credit vulnerability. Kozak, Aaron and Gauthier (2005) and Aaron and Hogg (2005) have shown the ability of the CCA to provide insights into the distribution of individual distance-to-default measures within a sector analysis.

## 2. Model framework

### 2.1. Model scope of application

The PD model is applicable to UK municipalities (e.g. Greater London Authority, Swindon Borough Council, Oxfordshire County Council). These entities are local authorities that share similar responsibilities of delivering public services supported directly and indirectly by taxes or government transfers. In this vein, the ability of an entity to increase taxed income and contain public spending is crucial. Non-UK entities should be out of the scope of this model, as different tax and bankruptcy laws apply across geographical jurisdictions (e.g. the USA). UK universities, although funded by government grants and fees, should fall under a separate PD model that considers different factors (e.g. applications per place, global ranking or fee-driven revenue). Sovereign entities and public bodies related to local authorities (e.g. NHS trusts) should also be given a separate treatment. Housing associations (e.g. UK Registered Social Landlords) that provide affordable social accommodation are out of the scope of the PD model. Finally, UK local authorities/government supported entities that are not municipalities (e.g. fire services, national parks, public high schools) are excluded from the scope, as the model factors are specifically designed for UK cities/counties. Nonetheless, prospective users of this model may wish to consider cascading the calculated PD estimates of the in-scope UK municipalities to specific entities under these local authorities. All in all, this model remains applicable to UK local and regional governments.

### 2.2. Key concepts

In developing the PD model for UK municipalities, we assume the following definitions:

1. Probability of default (PD). The likelihood that by the end of a specified time period (1 year) the entity under consideration would pass into default. At this point, we refer to the counterparty and not the particular obligation following recommendations made by the Basel Committee on Banking Supervision (see the impact study of BCBS 2002). Thus, we focus on modelling the counterparty's economic and financial conditions without discerning between the types of credit obligation.

2. Default. Choosing the definition of default presented in the Financial Conduct Authority's rules for the internal ratings based approach (see BIPRU 4.6.20 Article at FCA 2007), we note that the default occurs when either of the two scenarios happen:

– a counterparty is past due more than 90 calendar days on any credit obligation to a bank,

– a bank considers that a counterparty is unlikely to pay its debt in full, without recourse by the bank to actions such as realising collateral.

3. Distance-to-default. The measure of default happening when the value of an entity's assets falls below the default point (value of debt). We see it as the distance between the expected value of counterparty's assets and the default point.

4. Municipality. The local government or the local authority which delivers public services supported via taxes and transfers from other levels of government.

5. Conservatism. The approach to modelling that ensures that the model does not underestimate the probability of default.

## 3. Data

### 3.1. Summary of data

This section summarises the sourcing and preparation of the final dataset used for developing and validating the PD model. We collected complete financial and statistical data for 264 entities representing UK municipalities in 2013. Then, for 2014, we added eighteen more entities for which the financial data was not available in the previous year. Additional thirteen entities were added for the 2015 portfolio. The PD model was calculated for annual snapshots of each year to capture the changes to the stability of the underlying portfolio. In doing so, we could observe the migration of PD grades over time. In 2015, there were 433 local authorities (municipalities, districts, councils and boroughs) in the UK. The sample represents 68% of the UK municipalities for 2015. It should be noted that there are no defaults in the 3-year time-series for which we gathered financial and non-financial data.

The main source of data for the model input factors is the National Statistics website (www.statistics.gov.uk). For financial factors, we rely on the Statements of Accounts for UK local councils that are reported as consolidated accounts. These data sources contain the Revenue Account, the Balance Sheet and Notes that are relevant and audited data inputs. Figure 1 shows the uses of the data sources for the PD model and Table 3 provides the list of relevant sources.

In addition to the quantitative economic and financial data, we propose the use of qualitative data to assess the management quality of an entity. As shown in Table 4, the "entity governance" variable assigns different scores for all UK municipalities on the basis of the assessment of specific qualitative factors. However, for simplicity, credit providers may wish to assign a universal score across all entities within the scope of the PD model.

The high level overview of the input data is presented in Table 5 with the averaged values for the individual annual snapshots. Table 5 only shows changes in capital and operating revenues. Complementing this table, it should be noted that the capital expenditure has been gradually decreasing from GBP 65.3 million in 2013 to GBP 61.6 million in 2014 and GBP 54.9 million in 2015. Interestingly, the ability of the UK local governments to generate income through taxes has deteriorated slightly. At the same time, personnel charges (including welfare costs) have been reduced in order to generate budgetary savings.

Table 6 shows the final list of factors considered for the PD model. At this point, we decided not to include the tax income as a percentage of the national average, as well as the capital expenditure ratio in further analysis and restrict model inputs only to seven factors (F1–F7).

Overall, the input factors are not highly correlated. We decided to reject F8 "tax income as percent of national average" from further analysis, as the council tax income is already included in the analysis of the operating balance and the operating revenue. Moreover, this factor is UK-specific and underpinned by the data that is only available for the UK local authorities, which renders the model inadequate for application to municipalities incorporated in other countries by prospective users. However, users are strongly advised to consider adding this factor in the model re-development process. Section 9 discusses the benefits of measuring the ability of a municipality to raise taxed income.

There is a positive correlation between the entity's unemployment as percentage of national unemployment and the direct debt ratio, as well as the population ratio. Table 7 shows more details on how the calculated input factors are related.

We use ratios for the input factors instead of the absolute values, as it is assumed by Martín and Trujillo (2004) and Carey and Hrycay (2001) that the use of financial and economic ratios improves the accuracy of model inputs. Furthermore, Westgaard and Van der Wijst (2001) opt for the PD model based on a series of financial ratios as the optimal approach to entities that are not quoted on stock markets (e.g. municipalities). The most recent study of Cardoso et al. (2013) highlighted numerous advantages of using ratios and combinations of ratios for PD modelling. Table 8 provides summary statistics for the calculated input factors.

Summarising the above table, we observe outliers in the data that constitute extremely low or high values for some inputs. For example, the Rutland County Council displays the highest GVA *per capita* as percentage of national average, as this is England's smallest county. Addressing this issue, we use the 1st/99th percentile cut-off floor/cap for all calculated input factors.

Overall, we confirm the completeness of the data with 0% of values being flagged as missing. However, for future redevelopment of the PD model, we propose to use the 'median' value when replacing missing data points. We tested this method by removing some values and replacing them with the median and mean values and found the median approach to have minimal impact on the regression coefficient for the factor. However, during the model redevelopment, prospective users should note that the variables with different distributions may not be impacted in the same way as our dataset.

## 4. Model development

### 4.1. Model estimation

The PD model determines the credit risk of a municipality on the basis of the UK sovereign rating; the extent to which the government is able to support this entity in times of economic downturn; and the individual score for this municipality. In this setting, a cap is applied to the municipality that has a better credit risk grading than the sovereign. Figure 2 explains how the PD grading is anchored at the sovereign level. It should be noted that, in a pioneering attempt, this paper only introduces the concept of linking PDs to the level of state support for future studies in this domain. We assume that for the UK the level of support is constant through years 2014–2016. Correct quantification of the state support to municipalities for a given point in time should be made on a pool of countries assessed individually and mapped to a generic scale. Prospective users are encouraged to utilise the idea of introducing the state support to the PD equation in their own methodology of quantifying the extent to which governments are able to support local authorities and protect them from bankruptcies.

Application of the state support depends on the current economic situation, politics and the legal set-up of municipalities. We propose to set this scalar to 1 for simplicity. However, users of this model are advised to adjust this scalar to their needs. For example, the US cities will receive less state support in the face of bankruptcy and are more fiscally independent than the UK cities. There are also specific laws in certain US jurisdictions that facilitate easy filing for bankruptcy (e.g. US Code, Title 11, Chapter 9. Bankruptcy protection[1]). Such laws do not exist within the European Union and the UK. Section 9 of this paper discusses the benefits and constraints of factoring in the state support scalar to model estimation.

A visual inspection of Figure 2 reveals that the final PD of a municipality cannot have a better grading than the sovereign. This applies a conservative approach to the PD model in line with CRR regulations. We argue that implementing a country cap constitutes a sound assumption, as the United Kingdom cannot be more likely to default than its cities and local authorities. The sovereign external rating serves as a baseline for the creditworthiness of any UK city within the scope of the PD model.

### 4.2. Model formula

As explained in the previous section, we assume that the credit risk of a municipality is determined by the sovereign rating, the country-specific level of support and the individual, relative strength of a given municipality. Thus, the model formula is based on the following concept:

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[1] http://www.uscourts.gov/services-forms/bankruptcy/bankruptcy-basics/chapter-9-bankruptcy-basics.

$$R_{Mi} = R_{UK} + SS_{UK} \cdot X_{Mi(rel)} \tag{1}$$

where: $R_{Mi}$ is the final credit risk rating for a municipality, $M_i$ derived from the sovereign rating for the UK, $R_{UK}$ that serves as a proxy for the baseline creditworthiness of a municipality. The scalar $SS_{UK}$ measures the level of state support to local authorities and depends on the country-specific legal and institutional set-ups.

After determining the sovereign rating (baseline creditworthiness of a municipality), $X_{Mi(rel)}$ measures the individual credit risk strength of a municipality relative to the national average. Since the absolute level of $X_{Mi(rel)}$ is already incorporated in the sovereign rating, we are specifically looking into the relative individual strength. $X_{Mi(rel)}$ incorporates various macroeconomic, financial and management factors that are specific to a given municipality.

The formula for the PD model can be expressed in the distance-to-default space. The equation below shows the initial calculation formula for the PD model:

$$DD_{Mi(t)} = DD_{UK(t)} + SS_{UK(t)} \cdot DD_{rel(t)} \tag{2}$$

where $DD_{Mi(t)}$ constitutes the distance-to-default of a municipality, $M_i$ at time $t$. The index $i$ is assigned to flag individual municipalities.

This metric is a sum of the default distance implied from the UK's sovereign rating $DD_{UK(t)}$ at time $t$ and the function of the distance-to-default of a municipality $M_i$ relative to the UK sovereign $DD_{rel(t)}$ and the state support scalar $SS_{UK(t)}$. The relative distance-to-default $DD_{rel(t)}$ is scaled by the scalar $SS_{UK(t)}$ that assesses the state support ability in the UK at time $t$. The scalar $SS_{UK(t)}$ takes values from 0 to any capped value above 1 depending on the level of state support (see equation 3). However, for simplicity of calculating the PD estimates, we set the scalar to 1 across all entities. This is due to the fact that we look only at one country (UK) with similar state support across Wales, Scotland and England. This scalar comes helpful when differentiating between different $j$ countries or different levels of government supported entities (e.g. museums, regional governments and municipalities).

$$SS_{j(t)} = \begin{cases} > 1 & \text{for regions with lower state support} \\ < 1 & \text{for regions with higher state support} \end{cases} \tag{3}$$

$DD_{rel(t)}$ is derived from the following formula:

$$DD_{rel(t)} = \alpha_0 + \sum_{k=1}^{n} \alpha_k \cdot X_{Mi(t)} + \varepsilon \tag{4}$$

where $\alpha_k$ denotes model coefficient, $X_{Mi(t)}$ is based on different input factors, $X_{Mi(t)}$ is the individual credit risk strength of a municipality, $M_i$ measured by the municipality-specific factors (macroeconomic, financial and management) and indexed by $i$.

We note that the PD model can be extended beyond the UK municipalities and applied to a pool of countries:

$$DD_{Mi(t)} = DD_{SOVj(t)} + SS_{j(t)} \cdot DD_{rel(t)} \tag{5}$$

where $DD_{SOVj(t)}$ is the default distance implied from the sovereign rating for a country $j$ at time $t$.

The individual strength of a municipality $DD_{rel(t)}$ is determined by the entity-specific financial and non-financial (macroeconomic and qualitative) factors for which we collected complete data. The aspects of introducing state support to the PD equation are novel and prospective users who have access to data beyond UK municipalities are advised to derive the correct scale for the state support factor.

## 4.3. Agency rating mapping

Calculated probabilities of default using the formula specified in equations (1)–(4) are mapped to the external agency ratings. Thus, a grading scale can be created for all entities. Table 9 shows the mapping matrix that is based on external agency default data for years 1981–2014. As a result, the major financial crises are captured by the data to ensure the conservatism of the mapping. Furthermore, the mapping considers variations in default risk implied by the agency ratings for a broad asset classes that display increased default rates as compared to the sovereign entities and municipalities. Therefore, the mapped PD gradings are not too liberal in view of recently increased default experience across sovereigns and local authorities, as evidenced in the study by Standard & Poor's (2013).

No historical default events among UK municipalities and the lack of a large agency co-rated portfolio of local authorities makes it difficult to calibrate the PD model's parameters directly to defaults or external agency ratings. Thus, the prospective users are advised to follow an internal grade replication approach and estimate most of the model's parameters by means of a least squares fit to internal master grades transformed into the distance-to-default measures. The mapping used in this paper involves the reliance on the global PD curve from agency data. Thus, the curve reflects the smoothed long run average default experience for the financial and non-financial corporates globally. Therefore, applying the curve that contains default events and six major recession periods to UK municipalities with no default history and for a one-year horizon must be approved by the local regulator (e.g. PRA) to confirm this level of prudence. We rely on this mapping only for the reason of increasing the conservatism of the PD model and addressing the relevant PRA statements (SS12/13 10.13 (180))[2] that require PD models to estimate expected default probabilities for the portfolio by relying on a representative mix of good and bad economic periods, rather than simply taking the historic average of default rates actually incurred by the bank over a period of years.

Furthermore, CRR Articles (Art. 180.1(f) and Art. 178) require banks to carry out the mapping to external agency ratings in a way that is based on a comparison of internal rating criteria to the criteria used by an external organisation; and on a comparison of the internal and external ratings of any common obligors. Therefore, we validate the PD model by conducting a benchmarking exercise to a limited agency co-rated portfolio. Table 10 shows our mapping of the calculated PD estimates to the external agency ratings.

[2] Prudential Regulation Authority (2013), Counterparty credit risk, Supervisory Statement, SS12/13, Bank of England.

## 5. Estimation results

Table 11 summarises the final parameters relative to the entity-specific strength factors using 6 significant decimals. Equation (2) depicts the final model specification that serves to reconcile the final model parameters reported in Table 11. The model parameter standard errors are computed by the standard SAS OLS and NLP procedures.

Table 11 contains factor weights to mark the relative importance of individual factors in driving the PD estimates. At this point, factor weights $W_i$ are calculated as an approximation of the contribution of individual factors to the overall variation in model outputs on the basis of parameter coefficients and their standard deviations:

$$W_i = \frac{\left|Coeff_i\right| \cdot StdDev_i}{\sum_{j=1}^{n}\left|Coeff_j\right| \cdot StdDev_j} \tag{6}$$

We present the risk weights in an attempt to facilitate a scholarly discussion about the relevance of the financial and economic factors and to benefit prospective users that would have to resort to experts' opinions with respect to factor weights in their PD models. Thus, this paper gives a point of reference to the approximation of the contribution of individual factors to the overall variation in model outputs. At this point, we note that users of this model might benefit from replacing factor F6 "one-year operating balance as percent of operating revenue" with a new ratio of usable reserves to revenue. The new factor should also receive a small weight of 5%. Factor F3 of the GVA as the percentage of national average is expected to be very high in some cases of applying this model to large cities. We argue that this factor should not exceed 15% in its weight in the PD model due to a possible bias of distorting the true picture of economic situation in certain municipalities. Factor F3 can be distorted by the boroughs of London that artificially increase the GVA average in relation to other municipalities. The artificial increase can be explained by an example of a borough where a small cluster of financial institution drives the GVA, but the majority of the area remains deprived (e.g. Tower Hamlets). Therefore, as further justified in Section 9, factor F3 should be constrained. Moreover, before implementing this or any PD model, users and analysts should have a detailed knowledge of their credit portfolios with insights into the macro-financial dynamics around their exposures. As evidenced in the case of factor F3, knowing the underlying data is the key to successful modelling.

A visual inspection of Table 11 reveals that the weightings for F3 factors – "GVA as percent of national average" and F4 – "direct debt ratio" are considerably high. At this point, we performed several trials of different model outputs based on changes to F3's weight. We found out that reducing this weight to 15% has a minimal effect on the model adj-$R^2$ (which is reduced to 51.14%) and other factors' weights (e.g. F7 is increased to 6%; F6 is increased to 7%). Nonetheless, the final decision was not to change the weightings. We advise the prospective users of this model to adjust the weightings to their own needs and regulatory requirements. We discuss this issue in more detail in Section 9.

## 6. Core results

As expected, we observe an increased probability of default among UK municipalities due to the global economic downturn and the worsened financial standing of local authorities. The deteriorating economic performance of UK cities is reflected by the higher PD estimates. The mean calculated PD estimates call for a detailed monitoring of the financial standing of UK councils by central government bodies. If this trend continues in years 2016–2017, the central bank (Bank of England) should create special-purpose rescue funds that will be channelled to troubled municipalities. For commercial banks, the PD estimates reported below should constitute a warning signal about the increasing credit risk displayed by municipalities and local authorities.[3]

## 7. Model validation

### 7.1. Portfolio stability and backtesting

This section investigates PD migration over time. Given the size of the sample, we also confirm here that the sample portfolio remains representative for the whole UK. Although we do not report any defaults within the timeframe of the analysis, we observe a further deterioration of PD estimates for the underlying portfolio, as shown in Figure 3. This can be linked to the overall downgrade of the UK sovereign rating in 2014 and the fact that UK government reduced the funding to local authorities by 28% in years 2014–2015; see official reports by the Department for Communities and Local Government (2015) and the National Audit Office (2014). Furthermore, the National Audit Office has pointed to the signs of financial pressure across municipalities and local authorities in years 2014–2015.

Figure 3 reports a significant deterioration of PD estimates for UK municipalities. We note that the majority of PD estimates graded on the AA+ level in 2013 have been downgraded to AA- and A+ levels in 2014–2015. Against this backdrop, Table 13 provides a migration matrix to capture changes to default probabilities in more detail. The analysis is conducted only on those entities which had a PD estimate calculated on an ongoing basis for each consecutive year in the backtesting sample (2013–2015). Thus, the 31 entities newly added in 2014 and 2015 are excluded from the grading migration analysis, as probabilities of default for these municipalities have not been calculated in 2013.

A visual inspection of Table 13 reveals that there is only one instance of a grading upgrade from the A level in 2013 to the A+ level in 2015. For 264 entities which had a PD estimate calculated on an ongoing basis for each consecutive year in the backtesting sample (2013–2015). Table 13 reports 229 downgrades, which translate into 87% of UK municipalities displaying increased probability of default.

The average PD estimates are higher in 2015, meaning that the likelihood of UK municipalities going bankrupt has increased (see Table 14). There are no defaults in the backtesting sample to compare the accuracy of the predicted PD estimates. However, we report the juxtaposition of the predicted PD estimates to the realised defaults.

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3 Calculated probabilities of default using the formulas (2)–(4) are available on request.

## 7.2. Benchmarking to agency ratings

UK municipalities and local authorities are rarely externally rated. There are only several entities for which we could source external agency ratings. Since the co-rated portfolio consists of several municipalities, we present the entire benchmarking test in Table 15. The calculated PDs are taken for the cohorts corresponding to an external rating event. In doing so, we ensure that the implied agency grades match the same time and economic/financial conditions of the actual external rating events. In this vein, we note that benchmarking the latest cohort remains counterfactual, as most of the external ratings come from 2012–2013 and in many cases the entity was marked “not on watch” by the external rating agencies after the latest rating update. Therefore, the 2013 ratings may not be adequate for the 2015 cohort of calculated PD estimates.

The average notch difference of 1.29 suggests that model outputs are more conservative as compared to the external benchmarks. With this in mind, we conclude that the model does not under-predict the defaults of the entities within its scope. The differences to the external grades can be explained by the fact that the mapping matrix for the implied agency grades has been based on the external ratings for financial and non-financial companies. However, due to data availability (UK municipalities are rarely externally rated), there were no external benchmarks for municipalities/local authorities for which we could draw an appropriate mapping table. We also note that the model framework is specifically designed to make the calculated PD estimates conservative in line with the CRR/CRD IV requirements.

# 8. Regulatory compliance

This section tests the PD model’s compliance with the existing (CRR/CRD IV) and forthcoming (IFRS 9) regulations. Insights into the regulatory framework provided in this paper should help prospective users expand the application of the model beyond internal risk calculations to include the regulatory-prescribed materiality assessments of exposures for municipalities and local authorities. With this in mind, key regulatory requirements are discussed from the model implementation perspective. All in all, we note the recent departure of the regulatory focus from ensuring conservative modelling to promoting the accuracy of PD estimates.

## 8.1. CRR/CRD IV

Table 15 addresses relevant CRR Articles by showing whether our PD model is compliant with the existing rules or hindered by the compliance gap that prospective users should address. Summing up, the regulators demand that any PD model used to assess credit risk be consistently applied with accurate grades that allow credit risk differentiation across entities. The model must identify credit risk changes in a timely manner in order to prompt necessary actions. Furthermore, any credit risk rating methodology should be independently evaluated by the internal audit functions (BCBS 2015a).

### 8.2. IFRS 9

This section outlines the regulatory expectations specific to banks reporting under International Financial Reporting Standards (IFRS). It is limited to an overview of potential gaps and areas of non--compliance of our PD model in relation to IFRS 9 "Financial instruments". IFRS 9 replaces International Accounting Standard (IAS) 39 and is associated with the accounting treatment of financial assets and liabilities (BCBS 2015a; IFRS Foundation 2014).

To be fully compliant with the IFRS 9, the estimates produced by any PD model should be:

– unbiased – providing the most accurate and unbiased estimates of default probability;

– point in time – explaining a significant increase in credit risk based on the level of deterioration in the quality of fundamental factors assigned to an entity;

– forward looking – capturing future projections of macroeconomic factors;

– reflecting the term structure – estimating a term structure over the life of a specific transaction, exceeding the one year limit.

Although the aforementioned requirements are similar to the existing requirements for regulatory capital estimates (CRR), there are a number of important differences. From the regulatory capital perspective, for example, estimates should be conservative and are often subject to floors and adjustments. From the perspective of IFRS 9, the estimates should be truly unbiased. Thus, any regulatory floor applied to the model results in non-compliance with IFRS 9.

For the purpose of producing unbiased results, the PD model applies appropriate input variables that include the population ratio, the direct debt ratio as well as management quality factors. The PD estimates appear to be conservative as opposed to external agency ratings. However, there is no meaningful benchmarking analysis. Moreover, insufficient default data cannot disprove or prove the accuracy of model estimates. Therefore, we cannot determine whether the model is producing the best possible estimates that are compliant with the IFRS 9 rules.

Given that we have only three years of observations with no default events captured, the PD model does not capture any observable relationship between defaults and fluctuations in the credit cycle. Nonetheless, the model highlights the deterioration of the PD estimates in the aftermath of the global financial crisis and the ensuing recession. We cannot, at this point, argue that the PD model reflects point in time variations.

The PD model needs to incorporate forward looking information reflecting the length of the contractual period and including any extension options. We note that the likelihood of the default risk remaining stable over time is very low due to the further deterioration of economic conditions and idiosyncratic factors captured by the PD model. Changes in the input factors have a material impact on the term structure for PD estimates. Therefore, the model should be equipped with transition matrices sourced from vendors to determine future PDs. The prospective users should also attempt to incorporate credit cycle indices resulting from the base scenario into PD term structure over time.

## 9. Methodology weaknesses and suggested refinements

The PD model presented in this paper is not free of limitations and methodological weaknesses. This section advises on theoretical avenues that can be pursued by prospective users of this model to improve its accuracy and predictive powers.

Firstly, the model would benefit from embedding credit cycle indices in the methodology framework. However, we were unable to capture a distinctive credit cycle among municipalities to convert fundamental entity specific credit indicators into the PD estimates. Having no default experience, we could not link a default event to the reported changes in underlying macroeconomic and financial factors. Furthermore, we have not conducted the model calibration exercise to confirm that PDs are benchmarked to appropriate implied grades. However, the backtesting confirms that our model is conservatively calibrated. The prospective users are advised to accommodate any changes to model calibration based on any new defaults in a broader data sample of municipalities.

Secondly, there is no reliable approach to calculating the state support scalar that is currently set to 1. When applying the model to different countries, the scalar should be revised by prospective users. For example, US municipalities are more fiscally independent with local taxes generating budgetary income and the state support is limited in the case of bankruptcy. On the other hand, Japanese municipalities do not rely on tax-generated income, but receive strong sovereign support. The prospective users are advised to adjust the state support scalar to their modelling needs and future economic conditions. The users should note that changing the value of the scalar results in different levels of dispersion in PD estimates. Setting the scalar between 0 and 1 allows for less variability in PD estimates and possible non-compliance with CRR Article 170.1d. Furthermore, state support can take many forms that should be reflected in future refinements of the scalar. As it transpires, municipalities can be supported vertically (direct transfer of funds from the government) or horizontally (transfer of funds between local authorities/municipalities). All in all, introducing state support to the PD model represent a major source of inspiration for prospective users that can empirically test for regularities in the scalar applicable across different countries and entities.

Thirdly, in an attempt to make our PD model universal, we removed the factor variable that measures the ability of a municipality to generate income from the council tax. However, prospective users of this model are advised to retain a factor measuring fiscal flexibility of a municipality. The importance of including this factor in the model is further highlighted by the fact that external rating agencies are measuring the ability of municipalities and local authorities to raise taxes and decrease spending. At this point, the users should understand that municipalities in certain countries cannot raise additional taxes beyond certain thresholds and limits are applied to cutting the social expenditures. For simplicity, the prospective users of this model are advised to introduce a qualitative measurement of the fiscal flexibility for certain countries with basic categories of high, medium and low ratings on the ability to generate income from taxes.

As far as other input factors are concerned, we confirm that the macroeconomic factors (F1, F2, and F3) and factors measuring financial conditions (F4, F5, and F6) are reliable indicators for UK municipalities, as these were used in previous studies. However, the prospective users of the model should revise the F1 "population as percent of national" factor when extending the scope of the model to multiple counties. We have found F1 to distort the relative inter-country differences in municipalities. For instance, applying F1 in its current form to Ireland and USA would result in the Irish municipalities having artificially inflated ratios due to the small population of the country. On the other hand, the US cities would display lower ratios, as the US population size is substantially larger. To address this bias, the users are advised to redefine F1 as the percentage of the population of a typical city globally or within the country sample.

Finally, we propose to reduce the weight for the F3 "GVA per capita as percent of national average" factor to 15%. As shown in Table 18, this reduction has a minimal effect on the model adjusted-$R^2$ and the weights of other input factors. Prospective users of the PD model are advised to consider different weightings for model factors and further constraints on F3, such as assigning it a lower weight, depending on its appropriateness for measuring the economic standing of a municipality. With this in mind, we note that F3 does not reflect the economic disparities within a municipality. For instance, the London Borough of Tower Hamlets enjoys a very high ratio of F3 thanks to the Canary Wharf financial district. However, this municipality is the third most deprived local authority in England (Aldridge et al. 2015). Therefore, the users should not underestimate the importance of retaining the F7 "entity governance" factor when revising factor weightings.

## 10. Conclusions

We developed a PD model for UK municipalities that benefits the following groups of prospective users:

1. AIRB banks. The paper advises on ways of ensuring compliance with CRR and IFRS 9 regulatory requirements for the IRB evaluation methodologies in credit risk. The proposed model serves as a basis for future model redevelopments benefiting banks that utilise their own models for calculating credit risk parameters (PD/LGD models).

2. FIRB banks. The paper proposes modelled responses to gaining the regulatory approval for empirical models estimating PDs of individual groups of clients, namely municipalities. Subject to the suggested refinements, the proposed model can be adopted by prospective users.

3. Commercial banks. The paper advises on the overhaul of the scorecard-based approaches to calculating PD estimates. The proposed model can be used to replace the existing methodologies in order to generate more accurate estimates of default probabilities among municipalities.

4. Regulators. The proposed model can serve as a benchmarking tool used by local regulators and supervisory authorities to validate PD models developed internally by AIRB and FIRB banks for municipalities and other sub-sovereign entities.

5. Rating agencies. This paper has highlighted the fact that municipalities are rarely externally rated, which in turn impacts the quality of model backtesting required by the regulators. Rating agencies are advised to increase their rating actions for local authorities.

Upon model validation, we find out that there is a limited default history for a meaningful backtesting analysis. Thus, we argue that the conservatism of the PD estimates remains conceptual. In light of discussed methodological weaknesses, we have proposed several improvements to the model that can be utilised by prospective users.

The analysis of the PD model's regulatory compliance has revealed the existence of contradictory regulations that further complicate the implementation of any credit risk model. As shown in this paper, where a model is compliant with the CRR/CRD IV rules and ensures the conservatism of the estimates, it is not compliant with IFRS 9. As a result, dispensing with the conservative approach may lead to a build-up of credit risk that cannot be accurately captured. With this in mind, we argue that PD models should remain conservative so the banks can accumulate sufficient capital to cover the crisis-induced default exposures. The core purpose of any PD model is to lay foundations for calculations of necessary rescue funds and capital charges that can be utilised in an event of default. With limited default data availability and outdated external agency ratings, a PD model for sub-sovereign entities is likely to be too liberal, hence underestimating credit risk build-ups. As regards the case under scrutiny, the lack of historical default experience is evident for UK municipalities and necessitates a conservative approach to modelling default probabilities. Furthermore, we note that no forward looking indicator will compensate for the liberal risk estimates and there is no obvious way of ensuring truly accurate estimates of default probabilities without the historical observations. Therefore, addressing the regulatory bias, we recommend using two separate models:

– a conservative PD model for calculating regulatory capital (regulatory conservatism);

– a core PD model for reporting requirements under IFRS 9 and monitoring point in time variations in credit risk (accurate estimates without the influence of the regulatory conservatism).

Finally, the core results reported in this paper signal a deteriorating financial and economic conditions of UK municipalities and the increasing probability of default across UK cities. As shown in the PD migration matrix, the majority of the PD estimates graded on the AA+ level in 2013 have been downgraded by the PD model to AA- and A+ levels in years 2014–2015. Although the backtesting exercise has not revealed any new defaults, the overall deterioration of the PD estimates should raise concern. At this point, the overview of the data underlying the PD model indicates lower operating and capital revenues across UK municipalities. We also observe a 54% increase in the average direct debt level among UK local authorities in 2014. In addition to the overall deterioration of the PD estimates, there are worrying distress signals indicating a worsened economic and financial conditions of UK cities (BBC 2013; HM Treasury 2014). Although no tests for a possible non-linearity of the PD estimates' long-term path have been made and the analysed data encompass only three years of observations, regulators and central banks are advised to monitor the economic performance of sub-sovereign entities.

## References

Aaron M., Hogg D. (2005), The use of microdata to assess risks in the non-financial corporate sector, *Financial System Review*, December, Bank of Canada.

Aktug R.E. (2014), A critique of the contingent claims approach to sovereign risk analysis, *Emerging Markets Finance and Trade*, 50 (sup 1), 294–308.

Aldridge H., Born T.B., Tinson A., MacInnes T. (2015), *London's Poverty Profile 2015*, New Policy Institute.

Altman E. (1968), Financial ratios, discriminant analysis and the prediction of corporate bankruptcy, *Journal of Finance*, 23(4), 589–609.

BCBS (2002), *Quantitative impact study 3. Technical guidance*, Bank for International Settlements.

BCBS (2015a), *Guidance on credit risk and accounting for expected credit losses*, BCBS Paper, d350, Bank for International Settlements.

BCBS (2015b), *Standards. Revisions to the standardised approach for credit risk*, Bank for International Settlements.

BBC (2013), *Liverpool mayor claims "city could be bankrupt in two years"*, http://www.bbc.co.uk/news/uk-england-merseyside-24348712.

Beaver S.D., William H., Wolfson M.A. (1992), The role of market value accounting in the regulation of insured depository institutions, in: J.R. Barth, R. Dan Brumbagh Jr. (eds.), *The Reform of Federal Deposit Insurance*, Harper Collins.

Bielecki T.R., Rutkowski M. (2004), *Credit Risk: Modeling, Valuation and Hedging*, Springer-Verlag.

Black F., Cox J.C. (1976), Valuing corporate securities: some effects of bond indenture provisions, *Journal of Finance*, 31, 351–367.

Blancher N., Mitra S., Morsy H., Otani A., Severo T., Valderrama L. (2013), *Systemic risk monitoring ("SysMo") toolkit – a user guide*, IMF Working Paper, WP/13/168.

Brown J. (2016), Hull and Bradford among northern UK cities "struggling most" despite Northern Powerhouse pledge, *Yorkshire Post*, 29 February.

Cardoso V.S., Guimaraes A.L.S., Macedo H.F., Lima J. (2013), Assessing corporate risk: a PD model based on credit ratings, *ACRN Journal of Finance and Risk Perspectives*, 2(1), 51–58.

Carey M., Hrycay M. (2001), Parameterizing credit risk models with rating data, *Journal of Banking and Finance*, 25(1), 197–270.

Chan-Lau J.A., Sy A.N.R. (2006), *Distance-to-default in banking: a bridge too far?*, IMF Working Paper, 06/215, International Monetary Fund.

Crosbie P., Bohn J. (2003), *Modeling Default Ris*k, Moody's KMV Company.

De Nicolo G., Tieman A. (2006), *Economic integration and financial stability: a European perspective*, IMF Working Paper, 05/296, International Monetary Fund.

Department for Communities and Local Government (2015), *Provisional local government finance settlement: England, 2016 to 2017 and future years,* 17 December, Statement to the Parliament, https://www.gov.uk/government/collections/provisional-local-government-finance-settlement-england-2016-to-2017.

EBA (2015), *Results from the 2014 Low Default Portfolio (LDP) exercise*, European Banking Authority Report.

ECB (2005), *Financial Stability Review, June*, European Central Bank.

Falkenstein E.G., Boral A., Carly L.V. (2000), *RiskCalc for private companies: Moody's default model*, Moody's Investors Service.

FCA (2007), *BIPRU 4.6. The IRB approach: retail exposures*, *FCA Handbook,* Financial Conduct Authority.

Gornall W., Strebulaev I.A. (2015), *Financing as a supply chain: the capital structure of banks and borrowers*, Rock Center for Corporate Governance at Stanford University Working Paper, 166.

Governing Institute (2015), *Bankrupt cities, municipalities list and map*, http://www.governing.com/gov-data/municipal-cities-counties-bankruptcies-and-defaults.html.

Gray D.F., Malone S.W. (2008), *Macrofinancial Risk Analysis*, John Wiley & Sons.

Harada K., Ito T. (2010), *Did merger help Japanese mega-banks avoid failure? Analysis of the distance to default of banks*, NBER Working Paper, 14518, National Bureau of Economic Research.

HM Treasury (2014), *Chancellor: "We need a Northern powerhouse"*, The Rt Hon George Osborne MP Speech, https://www.gov.uk/government/speeches/chancellor-we-need-a-northern-powerhouse.

IFRS Foundation (2014), *IFRS 9 financial instruments*, International Accounting Standards Board

Iqbal N., Ali S.A. (2012), *Estimation of probability of defaults (PD) for low default portfolios: an actuarial approach*, Enterprise Risk Management Symposium Paper, April, http://www.ermsymposium.org/2012/otherpapers/iqbal-ali-paper-03-20-12.pdf.

Kealhofer S. (2003a), Quantifying credit risk I: default prediction, *Financial Analysts Journal*, 59(1), 30–44.

Kealhofer S. (2003b), Quantifying credit risk II: debt valuation, *Financial Analysts Journal*, 59(3), 77–92.

Kozak M., Aaron M., Gauthier C. (2005), Using the contingent claims approach to assess credit risk in the Canadian business sector, *Financial System Review*, December, Bank of Canada.

Krainer J., Lopez A. (2003), *Forecasting bank supervisory ratings using securities market information*, Federal Reserve Bank of San Francisco Paper, February.

Lewis J. (2012), A contingent claims approach to measuring insolvency risk: an empirical assessment of the impact of the global financial crisis on Jamaica and its financial sector, *Journal of Business, Finance & Economics in Emerging Economies*, 7(2), 1–22.

Marin J.L.M., Ponce A.T. (2005), Structural models and default probability: application to the Spanish stock market, *Investment Management and Financial Innovations*, 2, 18–29.

Medioli A., Van Praagh A., Tudela M. (2014), *U.S. municipal bond defaults and recoveries 1970–2013*, Moody's Report, 170048.

Merton R.C. (1974), On the pricing of corporate debt: the risk structure of interest rates, *Journal of Finance*, 49, 1213–1252.

Moore S. (2013), 20 cities that may face bankruptcy after Detroit, *Newsmax Magazine*, 6 August.

Nagel S., Purnanandam A. (2015), *Bank risk dynamics and distance to default*, Becker Friedman Institute for Research in Economics Working Paper, March, University of Chicago.

National Audit Office (2014), *The impact of funding reductions on local authorities*, Local Government Report by the Comptroller and Auditor General, National Audit Office.

Prorokowski L. (2011), Recovery from the current banking crisis: insights into costs and effectiveness of response regulations, *Qualitative Research in Financial Markets*, 3(3), 193–223.

Saldias M. (2012), *Systemic risk analysis using forward-looking distance-to-default series*, Banco de Portugal Working Paper, 16/2012.

Standard & Poor's (2013), *International local and regional governments default and transition study: 2012 saw defaults spike*, 28 March.

Tudela M., Medioli A., Van Praagh A. (2013), *U.S. municipal bond defaults and recoveries 1970–2013*, Moody's Report, 151936.

United Cities and Local Governments (2014), *BT global city leaders summit*, Conference Proceedings, International Festival for Business, http://www.liverpoolvision.co.uk/wp-content/uploads/2014/03/IFB-2014-Interim-Evaluation-Report.pdf.

Vasicek O. (1984), *Credit valuation*, Moody's KMV White Paper.

Westgaard S., Van der Wijst N. (2001), Default probabilities in a corporate bank portfolio: a logistic model approach, *European Journal of Operational Research,* 135, 338–349.

Zhang A. (2009), *Statistical methods in credit risk modelling*, University of Michigan PhD thesis, https://deepblue.lib.umich.edu/bitstream/handle/2027.42/63707/ajzhang_1.pdf.

Zielinski T. (2013), *Merton's and KMV models in credit risk management*, Studia Ekonomiczne. Zeszyty Naukowe Uniwersytetu Ekonomicznego w Katowicach, 127.

# Appendix

Table 1

Modelling approaches to credit risk

| Approach | Description | Advantages | Disadvantages |
|---|---|---|---|
| Structural models | Firm-value approach models measuring the ability of an obligor to meet the contractual debt that is determined by the obligor's asset value | Financial ratios are strong predictors of subsequent regulatory ratings (Krainer, Lopez 2003)<br>Useful in identifying risk factors that are emerging or difficult in quantification (Zhang 2009) | Merton-type contingent claims models produce PDs that are inconsistent with historically observed default rates (Falkenstein, Boral, Carly 2000)<br>Data requirements limit the scope of application for certain industries (Beaver, William, Wolfson 1992)<br>Dependence on distributional approaches (Zhang 2009) |
| Intensity based models | Reduced-approach models assuming that the default is determined solely by the asset value and that the default event is governed by an externally specified intensity process. The default is treated as a surprise event | Useful in simulating the future default intensity for the purpose of predicting the conditional default probability (Bielecki, Rutkowski 2004) | Lacking a comprehensive interpretation of a default event (Zhang 2009) |

Table 2

Summary of data

| Analysis | Year | Entities (number) | Defaults (number) | Data publication date* |
|---|---|---|---|---|
| Model development | 2013 | 264 | 0 | March 2014 |
| PD portfolio stability | 2014 | 282 | 0 | March 2015 |
| Benchmarking<br>Backtesting | 2015 | 295 | 0 | March 2016 |

* The financial and non-financial data for each analysed year is made publicly available in March of the next year. Data publication date is also the day when the data were extracted.

Table 3
Data sources for model inputs

| Input variable (factor) | Source | Description and use |
|---|---|---|
| Population entity | National statistics: local authority key statistics: population and vital statistics | The estimated population of a local authority is updated on an annual basis. It is used to calculate the population of an entity as percentage of national population (F1) |
| Unemployment rate | National statistics: unemployment statistics: unitary authority and local area district tables | This data is used to calculate the entity-specific unemployment as a percentage of national average (F2) |
| GVA (gross value added) | National statistics: sub-regional: gross value added | GVA (gross value added) is a well-recognised indicator of economic performance. It is used to calculate the 5-year GVA per capita as a percentage of the national average GVA per capita (F3) |
| Council tax | England – local ODPM: council taxes<br>Wales – Stats Wales: local government finance: council tax: levels: composition of average band D council tax by year<br>Scotland – Scotland: local government finance: council tax | The income. It is used to calculate the average tax of an entity as a percentage of the national average. Calibrated separately for England, Wales and Scotland (F8) |
| Entity governance | Qualitative query: review of information provided in general media and budget announcements by an entity | The qualitative information obtained from media (e.g. newspapers) and the assessment of financial information is used to assess the management quality (F7) |
| Direct debt | Statement of accounts: notes to the revenue accounts | Used to calculate direct debt/operating revenue ratio. The higher this ratio is, the more direct debt a local authority has, resulting in higher default probability (F4) |
| Debt service costs | Statement of accounts: consolidated balance sheet | Used to calculate debt service costs (interest expense) / operating revenue ratio (F4) |
| Operating revenue | Statement of accounts: revenue account | Used to calculate 1-year operating revenue growth. The higher the revenue growth of an entity the better the financial performance (F5) |
| Capital expenditure | Statement of accounts: balance sheet: notes attached to fixed assets | Used to calculate the capital expenditure as percentage of operating expenditure. The higher the ratio, the more an entity invests in capital (F9) |
| Personnel charges | Statement of accounts: balance sheet | Used to calculate the rigidity of structural expenses expressed as the ratio of personnel charges + interest paid + debt repayment to operating revenue. Personnel charges include welfare costs and pension contributions (F6) |

Table 4
Entity governance

| Score | Rationale |
|---|---|
| 9–10 | Management team with outstanding leadership qualities and extensive business experience<br>Statutory financial reporting is prepared on time and easily available<br>Budgets are clearly stated and split into capital and operating expenditure<br>There have been no significant overspends and there are no plans for abnormal expenditure<br>Suitable budget buffers are in place<br>There is a comprehensive, fully embedded risk management process<br>Environmental, social and governance (ESG) framework exists<br>Sustainability issues are properly addressed<br>Collection of revenue is timely and supported by a reliable council tax system |
| 7–8 | Management team with very good leadership qualities<br>Budgets are clearly stated and split into capital and operating expenditure and are easy to follow<br>There are only minor deviations over time from originally approved budgets<br>There is a comprehensive, fully embedded risk management process<br>Collection of revenue is timely and supported by a reliable council tax system |
| 5–6 | Management team enjoys a strong reputation in the region/sector<br>Budgets are clearly stated and split into capital and operating expenditure<br>Budget assumptions are realistic<br>There are small deviations over time from originally approved budgets<br>There is a comprehensive, fully embedded risk management process<br>Collection of revenue is timely and supported by a reliable council tax system |
| 3–4 | Quality of leadership is below average<br>Budgets are less clearly stated and goals are not explicitly translated into the budget<br>There are deviations over time from originally approved budgets without strong mitigation<br>Collection of revenues is below average for the region (England, Wales, Scotland)<br>Collection of revenue is timely and supported by a reliable council tax system |
| 1–2 | Quality of leadership is poor and its reputation damaged by scandals and financial embezzlements<br>There is no risk management process in place<br>There are abnormal budget expenditures |

Table 5
Overview of data

| Variables | 2013 | 2014 | 2015 |
|---|---|---|---|
| Entities (number) | 264 | 282 | 295 |
| Average population (number) | 249,660 | 239,287 | 240,605 |
| Average unemployment (%) | 6.78 | 6.78 | 6.78 |
| Average GVA (GBP) | 67,275 | 66,197 | 31,817 |
| Average tax (GBP) | 1,352 | 1,349 | 1,349 |
| Average governance score (1–10) | 6.68 | 6.68 | 6.68 |
| Average debt costs (GBP) | 13,635,528 | 13,311,673 | 12,172,750 |
| Average operating revenue (GBP) | 513,120,276 | 466,893,986 | 452,765,222 |
| Average personnel charges (GBP) | 47,128,614 | 41,941,959 | 40,830,769 |
| Average interest expense (GBP) | 4,384,839 | 4,832,345 | 3,996,317 |
| Average long term debt (GBP) | 199,806,370 | 198,158,748 | 192,712,996 |
| Average capital revenue (GBP) | 11,726,581 | 10,804,699 | 5,490,915 |

Table 6
Final list of calculated factors

| Data source | Final input factor | Factor description |
|---|---|---|
| National statistics | F1 | Population as percent of national |
| | F2 | Unemployment as percent of national average |
| | F3 | GVA per capita as percent of national average |
| Statements of accounts | F4 | Direct debt ratio |
| | F5 | One-year operating revenue growth |
| | F6 | One-year operating balance as percent of operating revenue |
| Qualitative | F7 | Governance |
| Regional statistics | F8 | Tax income as percent of national average – not included in further analysis |
| Notes to fixed assets | F9 | Capital expenditure as percent of operating expenditure – not included in further analysis |

Table 7
Pearson correlation matrix (calculated model inputs)

| Pearson coefficient | F1 | F2 | F3 | F4 | F5 | F6 | F7 |
|---|---|---|---|---|---|---|---|
| F1 | 1 | | | | | | |
| F2 | 0.227 | 1 | | | | | |
| F3 | 0.111 | 0.064 | 1 | | | | |
| F4 | 0.270 | 0.350 | 0.014 | 1 | | | |
| F5 | 0.065 | -0.042 | 0.066 | -0.119 | 1 | | |
| F6 | 0.023 | -0.034 | 0.002 | -0.001 | 0.102 | 1 | |
| F7 | -0.062 | -0.065 | 0.132 | 0.056 | 0.022 | 0.106 | 1 |

Table 8
Summary statistics for calculated input factors

| Annual snapshot | Input factor | N | Mean | Median | Std. dev. | Min | Max |
|---|---|---|---|---|---|---|---|
| | F1 | 264 | 1.052 | 1.041 | 0.067 | 0.864 | 1.253 |
| | F2 | 264 | 0.956 | 0.965 | 0.305 | 0.173 | 1.400 |
| | F3 | 264 | 15.009 | 0.816 | 109.086 | 0.001 | 997.568 |
| 2013 | F4 | 264 | 0.430 | 0.340 | 0.427 | 0 | 2.434 |
| | F5 | 264 | 0.049 | -0.014 | 0.807 | -0.999 | 8.898 |
| | F6 | 264 | -0.098 | 0.0003 | 0.476 | -2.164 | 0.878 |
| | F7 | 264 | 6.684 | 6.285 | 1.156 | 4 | 10 |
| | F1 | 282 | 1.036 | 1.035 | 0.108 | 0.685 | 1.384 |
| | F2 | 282 | 0.958 | 1 | 0.280 | 0.288 | 1.400 |
| | F3 | 282 | 19.733 | 0.858 | 122.780 | 0.003 | 913.037 |
| 2014 | F4 | 282 | 0.662 | 0.509 | 0.564 | 0 | 2.117 |
| | F5 | 282 | 0.099 | -0.023 | 0.987 | -0.904 | 8.661 |
| | F6 | 282 | -0.143 | 0 | 1.020 | -9.139 | 0.888 |
| | F7 | 282 | 6.665 | 6.405 | 1.188 | 4 | 10 |
| | F1 | 295 | 1.040 | 1.037 | 0.005 | 0.685 | 1.384 |
| | F2 | 295 | 0.948 | 0.972 | 0.299 | 0.173 | 1.400 |
| | F3 | 295 | 17.007 | 0.868 | 115.100 | 0.001 | 1.007.335 |
| 2015 | F4 | 295 | 0.515 | 0.376 | 0.048 | 0 | 11.339 |
| | F5 | 295 | 0.040 | -0.014 | 0.046 | -0.999 | 8.961 |
| | F6 | 295 | -0.080 | 0.0002 | 0.038 | -9.146 | 0.999 |
| | F7 | 295 | 6.454 | 6 | 0.067 | 4 | 10 |

Table 9
Agency default rates observations for ratings (1981–2014)

| S&P and Moody's grades | Non-financial companies | | | | Financial companies | | | |
|---|---|---|---|---|---|---|---|---|
| | S&P | | Moody's | | S&P | | Moody's | |
| | cohort observa-tions | default rate (%) | cohort observa-tions | default rate (%) | cohort observa-tions | default rate (%) | cohort observa-tions | default rate (%) |
| AAA<br>Aaa | 1,391 | 0.000 | 1,462 | 0.000 | 2,132 | 0.000 | 1,718 | 0.000 |
| AA+<br>Aa1 | 752 | 0.000 | 1,037 | 0.000 | 1,079 | 0.000 | 1,651 | 0.000 |
| AA<br>Aa2 | 2,925 | 0.000 | 1,635 | 0.000 | 2,806 | 0.036 | 2,271 | 0.000 |
| AA-<br>Aa3 | 2,654 | 0.000 | 2,829 | 0.000 | 3,606 | 0.055 | 3,694 | 0.108 |
| A+<br>A1 | 4,196 | 0.048 | 4,203 | 0.000 | 4,223 | 0.071 | 3,524 | 0.199 |
| A<br>A2 | 7,233 | 0.014 | 6,246 | 0.048 | 4,680 | 0.171 | 3,241 | 0.062 |
| A-<br>A3 | 6,378 | 0.031 | 6,494 | 0.031 | 4,077 | 0.147 | 2,675 | 0.112 |
| BBB+<br>Baa1 | 6,999 | 0.114 | 6,568 | 0.107 | 2,797 | 0.107 | 1,631 | 0.307 |
| BBB<br>Baa2 | 8,815 | 0.147 | 7,447 | 0.081 | 2,559 | 0.313 | 1,547 | 0.517 |
| BBB-<br>Baa3 | 6,601 | 0.227 | 6,120 | 0.278 | 1,960 | 0.561 | 1,162 | 0.172 |
| BB+<br>Ba1 | 4,151 | 0.193 | 3,597 | 0.528 | 1,041 | 0.865 | 829 | 0.724 |
| BB<br>Ba2 | 5,505 | 0.636 | 3,831 | 0.444 | 932 | 0.429 | 602 | 0.498 |
| BB-<br>Ba3 | 7,421 | 1.118 | 5,650 | 1.611 | 882 | 1.134 | 655 | 2.595 |
| B+<br>B1 | 10,223 | 2.191 | 6,975 | 2.065 | 806 | 1.737 | 582 | 2.405 |
| B<br>B2 | 7,630 | 4.351 | 6,821 | 3.357 | 653 | 2.757 | 416 | 2.404 |
| B-<br>B3 | 3,595 | 8.039 | 6,948 | 4.735 | 463 | 3.240 | 324 | 6.481 |
| CCC+<br>Caa1 | 1,177 | 20.986 | 3,618 | 5.998 | 126 | 8.730 | 126 | 7.143 |
| CCC<br>C | 1,240 | 34.435 | 4,341 | 17.047 | 166 | 19.880 | 326 | 15.644 |

Source: S&P CreditPro (January 1981 – December 2014); Moody's DRS (January 1981 – December 2014).

Table 10
Agency alphabet grading

| Calculated PD (%) | S&P | Moody's |
|---|---|---|
| 0.000– 0.005 | AAA | Aaa |
| 0.006–0.015 | AA+ | Aa1 |
| 0.016–0.025 | AA | Aa2 |
| 0.026–0.035 | AA- | Aa3 |
| 0.036–0.055 | A+ | A1 |
| 0.056– 0.070 | A | A2 |
| 0.071–0.095 | A- | A3 |
| 0.096–0.135 | BBB+ | Baa1 |
| 0.136–0.205 | BBB | Baa2 |
| 0.206–0.315 | BBB- | Baa3 |
| 0.316–0.500 | BB+ | Ba1 |
| 0.501–0.810 | BB | Ba2 |
| 0.811–1.375 | BB- | Ba3 |
| 1.376–2.350 | B+ | B1 |
| 2.351–4.155 | B | B2 |
| 4.156–7.590 | B- | B3 |
| 7.591–14.840 | CCC+ | Caa1 |
| 14.841–27.900 | CCC | C |

Table 11
PD model parameters

| Factor | Units | Parameter $\alpha$ | Median | Standard error | t value | Pr > \|t\| | Weight (%) |
|---|---|---|---|---|---|---|---|
| Intercept | – | -0.420 | n/a | 0.019 | -21.150 | < 0.0001 | – |
| F1 | ratio | 0.141 | 1.058 | 0.017 | 4.820 | < 0.0001 | 4 |
| F2 | ratio | -0.090 | 0.952 | 0.005 | -18.950 | < 0.0001 | 18 |
| F3 | ratio | 0.091 | 0.932 | 0.005 | 29.550 | < 0.0001 | 26 |
| F4 | ratio | -0.113 | 0.324 | 0.003 | -33.600 | < 0.0001 | 31 |
| F5 | ratio | 0.154 | 0.053 | 0.010 | 14.110 | < 0.0001 | 13 |
| F6 | ratio | 0.030 | -0.0002 | 0.004 | 5.850 | < 0.0001 | 5 |
| F7 | qualitative scale | 0.014 | 6.460 | 0.002 | 3.270 | < 0.0001 | 3 |

Notes: adjusted R-squared: 54.85%, observations 841.

Table 12
Mean calculated PDs

| Year | Count observations | Mean calc. PD (%) | Median PD (%) |
|---|---|---|---|
| 2013 | 264 | 0.016 | 0.014 |
| 2014 | 282 | 0.032 | 0.029 |
| 2015 | 295 | 0.055 | 0.037 |

Table 13
PD estimates migration matrix (2013 vs. 2015)

| | | **2015 implied gradings** | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | AAA | AA+ | AA | AA- | A+ | A | A- | BBB+ | BBB | BBB- | BB+ | BB | BB- | B+ | Total |
| **2013 implied gradings** | AAA | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | **0** |
| | AA+ | 0 | 23 | 3 | 70 | 89 | 10 | 1 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | **198** |
| | AA | 0 | 0 | 0 | 5 | 14 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | **21** |
| | AA- | 0 | 0 | 0 | 11 | 18 | 10 | 0 | 2 | 0 | 0 | 0 | 0 | 0 | 0 | **41** |
| | A+ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | **0** |
| | A | 0 | 0 | 0 | 0 | 1 | 3 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | **4** |
| | A- | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | **0** |
| | BBB+ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | **0** |
| | BBB | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | **0** |
| | BBB- | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | **0** |
| | BB+ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | **0** |
| | BB | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | **0** |
| | BB- | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | **0** |
| | B+ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | **0** |
| | Total | **0** | **23** | **3** | **86** | **122** | **24** | **2** | **3** | **0** | **0** | **1** | **0** | **0** | **0** | |

Table 14
Predicted vs realised PDs

| Year | Count observations | Mean calc. PD (%) | Mean realised PD (%) |
|---|---|---|---|
| 2013 | 264 | 0.016 | 0 |
| 2014 | 282 | 0.032 | 0 |
| 2015 | 295 | 0.055 | 0 |

Table 15
Benchmarking analysis

| Entity name | Calc. PD (%) at a time of rating event | S&P implied grade | MD implied grade | External rating | Latest rating update | Notch difference |
|---|---|---|---|---|---|---|
| Cornwall Council | 0.015 | AA+ | Aa1 | Aa1 | 25 Feb 2013 | 0 |
| Birmingham City Council | 0.031 | AA- | Aa3 | AA+ | 17 Jul 2012 | 2 |
| Wandsworth Borough Council | 0.015 | AA+ | Aa1 | AA+ | 9 Jan 2013 | 0 |
| Greater London Authority | 0.028 | AA- | Aa3 | AA+ | 30 Jun 2014 | 2 |
| Guildford Borough Council | 0.029 | AA- | Aa3 | Aa1 | 25 Feb 2013 | 2 |
| Lancashire County Council | 0.039 | A+ | A1 | Aa2 | 25 Feb 2015 | 2 |
| Warrington Borough Council | 0.035 | AA- | Aa3 | Aa2 | 20 May 2015 | 1 |

Notes: notch difference = implied grade – agency rating; positive values suggest that the model is conservative; negative values inform about the model being too liberal.

Table 16
CRR compliance checks

| CRR article | Text | Status | Comments |
|---|---|---|---|
| 160.1 | The calculated PD of an exposure to a corporate institution cannot be lower than 0.03% | Not applicable | The floor of 0.03% is not applied to the PD model, as it deals with sub-sovereign entities (municipalities). No corporate institution is in the underlying portfolio |
| 170.1d | Portfolios concentrated in a particular market segment and range of default risk shall have enough obligor grades within that range to avoid undue concentrations of obligors in a particular grade. Significant concentrations within a single grade shall be supported by convincing empirical evidence that the obligor grade covers a reasonably narrow PD band and that the default risk posed by all obligors in the grade falls within that band | Compliant | Troubled municipalities have been added to the portfolio to ensure that the model produces sufficient discrimination across the portfolio with a wide range of calculated PDs and implied grades |
| 174.1a | If an institution uses statistical models and other mathematical methods to assign exposures to obligors or facilities grades or pools, the model shall have good predictive power and capital requirements shall not be distorted as a result of its use. The input variables shall form a reasonable and effective basis for the resulting predictions | Gap | Given limited observations with no default events captured, it is difficult to assess whether the model has good predictive powers. Input variables encompass the financial and macroeconomic indicators and form a reasonable basis for the resulting predictions |
| 174.1b | If an institution uses statistical models and other mathematical methods to assign exposures to obligors or facilities grades or pools, the institution shall have in place a process for vetting data inputs into the model, which includes an assessment of the accuracy, completeness and appropriateness of the data | Compliant | The model uses complete data from reliable sources. The accuracy of the model inputs is additionally ensured by using ratios |
| 174.c | The data used to build the model shall be representative of the population of obligors or exposures | Compliant | The data consists of municipalities from Scotland, Wales, England. Additionally, troubled municipalities from the most deprived regions are added to extend the scope of the model and introduce wider spreads of calculated PD values |

| | | | |
|---|---|---|---|
| 180.1a | Institutions shall estimate PDs by obligor grade from long run averages of one-year default rates.<br>PD estimates for obligors that are highly leveraged or for obligors whose assets are predominantly traded assets shall reflect the performance of the underlying assets based on periods of stressed volatilities | Compliant | PD values are based on 1-year estimates. Moreover, assets of the municipalities are rarely traded or highly leveraged |
| 180.1d | In quantifying the risk parameters to be associated with rating grades or pools, institutions shall use PD estimation techniques only with supporting analysis. Institutions shall recognise the importance of judgmental considerations in combining results of techniques and in making adjustments for limitations of techniques and information | Compliant | Appropriate section contains judgmental considerations of the PD estimation techniques and discusses limitations of information/data |

Table 17
IFRS 9 compliance checks

| IFRS 9 category | Status | Comments |
|---|---|---|
| Unbiased | Not determined | The PD estimates are conservative. However, there is not enough default and external benchmarking data to either prove or disprove the bias towards conservatism. Therefore, the model is deemed to provide the best possible estimates of default probability |
| Point in time | Not determined | The model is based on a wide range of inputs that provide information on population ratio, unemployment, direct debt load and financial standing. The input factors reflect changes to the default risk over time. However, the risk weights do not vary over time and the three observations provide limited evidence for the point-in-time PDs |
| Forward looking | Not determined | The model is designed to serve as an early warning system for the build--up of credit/default risk in the sub-sovereign sector. It signals the worsening economic conditions of the public bodies. However, model factors do not seem to capture future projections of macroeconomic conditions |
| Term structure | Not compliant | Although the PD model is theoretically capable of incorporating forward looking information, this is not used for the current version. A transition matrix is not available for this model |

Table 18
Revised estimation results (F3 weight set to 15%)

| Factor | Description | Estimate | Weight (%) |
|---|---|---|---|
| n/a | Intercept | -0.420 | n/a |
| F1 | Population as percent of national | 0.141 | 8 |
| F2 | Unemployment as percent of national average | -0.090 | 18 |
| F3 | GVA as percent of national average | 0.091 | 15 |
| F4 | Direct debt ratio | -0.113 | 32 |
| F5 | One-year operating revenue growth | 0.154 | 14 |
| F6 | One-year operating balance as percent of operating revenue | 0.030 | 7 |
| F7 | Governance | 0.014 | 6 |

Figure 1
Data flow chart (model development)

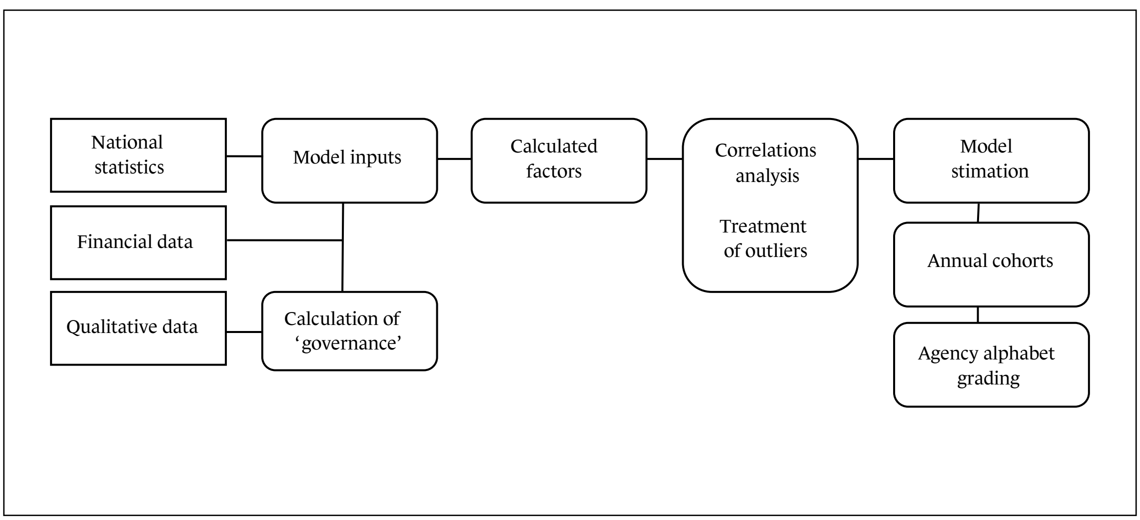

Figure 2
Model calculation process

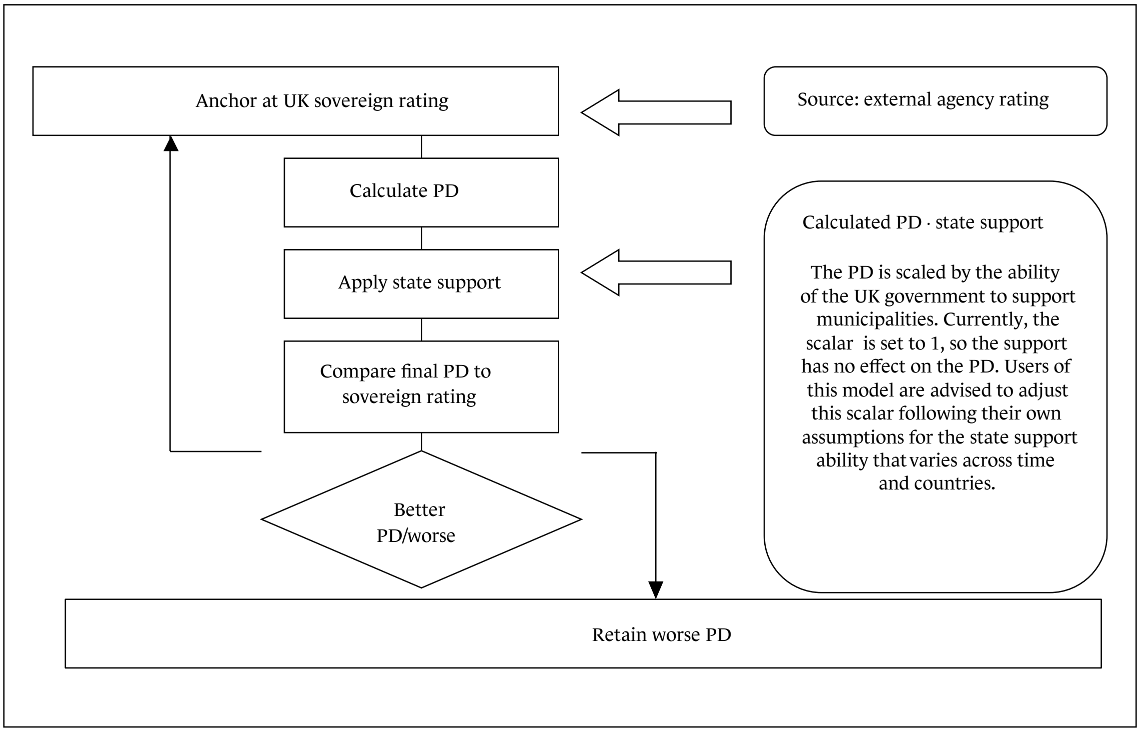


Figure 3
PD estimates distribution (implied gradings)